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# Quantitative analysis for acoustic characteristics of porous metal materials by improved Kolmogorov's turbulence theory

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ABSTRACT

This paper investigates the sound absorbing property of porous metal materials quantitatively using turbulence analogy model. Firstly, the improved Kolmogorov's turbulence theory is obtained by catastrophe theory for the first time, to our knowledge. Secondly, a quantitative turbulence analogy model is proposed though the improved Kolmogorov's turbulence theory. Finally, this model is adopted to analyze the wave propagation inside the porous metal materials. The quantitative relationship of energy spectrum density is fully obtained, which is also related to the average pore diameter and porosity. With the increase of excitation frequency, the energy spectrum density decreases with certain power law. And the energy increases with the increase of the average pore diameter. Our theoretical results are consistent with the experimental results. Our study can provide a feasible theoretical guidance for controlling the vibration and noise of porous metal materials.

## 1. Introduction

Porous metal materials are widely applied in noise control in various industries due to their advantages in heat resistance, lightness, and stiffness and so on [1–3]. In recent years, with the development of porous metal material preparation process, the acoustic material with excellent sound absorption characteristics can be obtained by structural optimization [4].

Biot presented the constitutive and fluctuation control equations of porous media for analyzing the acoustical properties in 1956 [5,6]. In 1992, the equivalent fluid model has been found by Allard, which considered the impact of the air viscosity, the heat conduction, and the structure factor on acoustic wave propagation [7]. More recently, Xu et al. found that local turbulence effect and volume dissipation are two important reasons for the transformation of kinetic energy to heat in porous materials under the excitation of acoustic [8]. Hu et al. studied the energy dissipation process of the porous materials by the turbulent like method, and pointed out that the energy spectral density of sound waves inside porous materials is proportional to the $-5/3$ power of the wave number under high frequency acoustic excitation [9]. Zhang et al. and Wang et al. investigated the sound absorption properties of porous materials below 150 dB though numerical simulation and experiments [10,11]. Wu et al. presented a quantitative theoretical model to investigate the sound absorbing property of metal rubber with high temperature and high sound pressure based on Kolmogorov's turbulence theory, and showed that the sound pressure amplitude of the porous metal material increases with increasing temperature and sound pressure level [9]. Although these acoustical models could describe the absorption properties of porous metal materials, unfortunately it cannot explain the mechanism of energy dissipation. And, due to the limitation of Kolmogrov's turbulence theory, the mechanisms of the sound absorption of porous metal materials under acoustic excitation have been still not fully understood.

In this paper, a turbulence analogy model is proposed by improved Kolmogorov's turbulence theory to investigate the sound absorbing property of porous metal materials quantitatively. This paper is organized as follows: In Section 2, the improved Kolmogorov's turbulence theory is studied by catastrophe theory. For analyzing the sound absorbing property of porous metal materials, the turbulence analogy model is presented in details in Section 3, and the numerical results and analysis are given in Section 4.

## 2. Quantitative analysis of the acoustic properties of porous metal materials by improved Kolmogorov's theory

The air particles inside the porous materials are in the turbulence state when the porous material is excited by acoustic wave [9], and the motion state of air particles transfers from turbulence phase to fully turbulent phase gradually with the increase of excitation frequency. Therefore, the acoustic properties of porous metal materials can be analyzed by turbulence analogy model.

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### 2.1. Quantitative analysis of the turbulent phase transition by catastrophe theory

Turbulence produces the vortices of many different length scales. Most of the kinetic energy of the turbulent motion is contained in the large-scale vortices. The energy transfer from the largest scale $L$ vortices to the smaller scales vortices $l$ by an inertial and essentially inviscid mechanism. This process continues, producing a hierarchy of vortices. Eventually this process creates the smallest vortices that the viscosity of the fluid can effectively dissipate the kinetic energy into internal energy [12].

Kolmogorov introduced the wave number $k(l = 1/k)$ to instead of scale $l$, and pointed out that the energy spectral density $E(k)$ is proportional to the $-5/3$ power of the wave number $k$ in fully turbulence state [13]. However, the whole process of turbulent phase transition cannot be solved yet.

Therefore, the process of turbulent phase transition can be revealed by the scale variation of the vortices. The potential function $V(x) = x^4 + tx^2 + ux$ [14] of the cusp catastrophe mode of the catastrophe theory can describe the scale change rules of turbulent phase transition. Where $x$ is a state variable, $t$ and $u$ are control variables. Taking the wave number $k(l = 1/k)$ as the state variable, the equilibrium surface equation that is the first derivative of potential function, can be expressed as:

$$k^3 + tk + u = 0 \tag{1}$$

According to Eq. (1), the equilibrium surface shows the rules of phase transition, as shown in Fig. 1. When the control variable $t < 0$, a point $P$ on the equilibrium surface jumps from lower sheet to upper sheet that represent different phases, meanwhile, the turbulent phase transition appears [14].

We analyze the control variables $t$ and $u$ by the dimensionless analysis, and suppose that the control variables $t$ and $u$ are described by the energy dissipation rate $\varepsilon$, the kinematic viscosity $\mu$, the density ρ, and the energy spectral density $E$, with $\alpha_1$, $\alpha_2$, $\alpha_3$, and $\alpha_4$ denoting the power exponents of $\varepsilon$, $\mu$, ρ, and $E$, respectively, thus $t$ and $u$ can be expressed as:

$$\begin{cases} t = A\varepsilon^{\alpha_1}\mu^{\alpha_2}\rho^{\alpha_3}E^{\alpha_4} \\ u = B\varepsilon^{\alpha_1}\mu^{\alpha_2}\rho^{\alpha_3}E^{\alpha_4} \end{cases} \tag{2}$$

We use three basic dimensions, time $T$, length $L$, and mass $M$ to describe the relationship among the power exponents. Because the dimension of $k$ is $[L^{-1}]$, the dimensions of $t$ and $u$ should be $[L^{-2}]$ and $[L^{-3}]$ to satisfy Eq. (1), and the relationships among the power exponents by the dimensionless analysis are listed in Table 1.

The power exponents must satisfy the following relationship

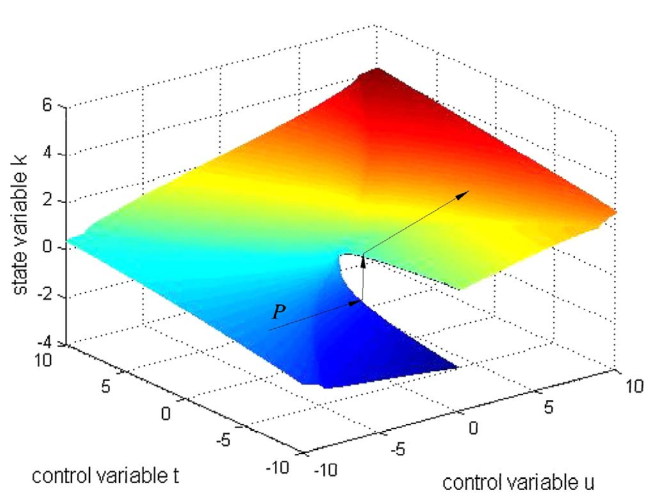


**Fig. 1.** The equilibrium surface of cusp catastrophe model.

**Table 1**
Relationships among the power exponents.

| | ε ($\alpha_1$) | μ ($\alpha_2$) | ρ ($\alpha_3$) | $E$ ($\alpha_4$) |
|---|---|---|---|---|
| $L$ | 2 | 2 | −3 | 2 |
| $T$ | −3 | −1 | 0 | 3 |
| $M$ | 0 | 0 | 1 | 1 |

$$\begin{cases} 2\alpha_1 + 2\alpha_2 - 3\alpha_3 + 2\alpha_4 = -2 \\ -3\alpha_1 - \alpha_2 + 3\alpha_4 = 0 \\ \alpha_3 + \alpha_4 = 0 \end{cases}$$
$$\begin{cases} 2\alpha_1 + 2\alpha_2 - 3\alpha_3 + 2\alpha_4 = -3 \\ -3\alpha_1 - \alpha_2 + 3\alpha_4 = 0 \\ \alpha_3 + \alpha_4 = 0 \end{cases} \tag{3}$$

From Eqs. (2) and (3), the control variables $t$ and $u$ are obtained as:

$$\begin{cases} t = A\varepsilon^{(2-\alpha_4)/4}\mu^{(-6-9\alpha_4)/4}\rho^{(-\alpha_4)}E^{\alpha_4}; \\ u = B\varepsilon^{(3-\alpha_4)/4}\mu^{(-9-9\alpha_4)/4}\rho^{(-\alpha_4)}E^{\alpha_4}; \end{cases} \tag{4}$$

where $A$ and $B$ are constants, and the range of $A$ and $B$ should be $A < 0$ and $0 < B < 1$, according to the property of cusp catastrophe model [14].

On substituting Eq. (4) into Eq. (1), the energy spectral density $E$ has the form:

$$E = -\rho\left(A\varepsilon^{\frac{(2-\alpha_4)}{4}}\mu^{\frac{-3(2+3\alpha_4)}{4}}k^{-2} + B\varepsilon^{\frac{(4-\alpha_4)}{4}}\mu^{\frac{-3(3+3\alpha_4)}{4}}k^{-3}\right)^{-1/\alpha_4} \tag{5}$$

Though analyzing the cusp catastrophe model [14], the two stable extreme points corresponding to two phases of turbulence, the turbulence phase and the fully turbulence phase, which are $\alpha_4 = -9/5$, and $\alpha_4 = -6/5$ (B = 0) respectively. On substituting $\alpha_4 = -9/5$ and $\alpha_4 = -6/5$ (B = 0) into Eq. (5), there are:

$$E = \rho\mu\varepsilon^{2/3}k^{-5/3}(B + A\mu^{3/4}\varepsilon^{-2/4}k^{1})^{5/9} \tag{6}$$

$$E = A\rho\mu\varepsilon^{2/3}k^{-5/3} \tag{7}$$

Eq. (6) shows the quantitative relationship of the turbulence phase, and Eq. (7) expresses the quantitative relationship for the fully turbulence phase which is in accordance with "−5/3th power" law of Kolmogorov's theory.

According to Eq. (5), Fig. 2(a) shows the relationship among the energy spectral density $E$, coefficient $\alpha_4$ and the wave number $k$. Fig. 3(b) shows the relationship between the energy spectral density $E$ and coefficient $\alpha_4$. In the range of $-2 < \alpha_4 < -9/5$ (region I) also called energy zone, with the increase of velocity, the energy of the fluid increases gradually due to the energy accumulation, and when $\alpha_4 = -9/5$, the velocity fluctuation appears firstly in turbulence phase, which forms the vortices at the largest scale $L$ and cause the transferring from laminar to turbulence. In the range of $-9/5 < \alpha_4 < -6/5$ (region II) also called dissipation zone, the energy gradually decreases, the largest scale $L$ vortices splits into smaller ones, meanwhile the energy also transfers to vortices of smaller scale $l$. And when $\alpha_4 = -6/5$, the energy transfers to the smallest scale $l_0$ vortices, and in the range of $\alpha_4 \leqq -6/5$ (region III) also called equilibrium zone, the energy is dissipated by viscous motion completely in fully turbulent phase.

### 2.2. Quantitative analysis of the acoustic properties of porous metal materials by turbulence analogy model

We analyze the energy spectrum density inside the porous metal materials by turbulence analogy method. Firstly, by the structural characteristics of porous metal materials, we suppose that the air in pores inside the porous metal material is equivalent to the particles of the particle flow, the metal skeleton is equivalent to the air in the particle flow, and the porosity of the porous metal material is