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The accuracy of some models for the airflow resistivity of nonwoven materials

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ABSTRACT

The airflow resistivity is a key parameter to consider when evaluating the acoustic performance of a fibrous material. The airflow resistivity is directly linked to a fibrous materials acoustic properties which allows for the non-invasive measurements of the fibre diameter and material density from acoustical data. There are several models that relate the airflow resistivity to the acoustic behaviour through the material's density and fibre diameter. It is not always obvious how accurately a model represents the true value of the flow resistivity of a nonwoven material with a fibre size variation. Therefore, the scope of this paper is to compare the performance of several theoretical and empirical models applied to a representative range of nonwoven fibrous media composed of blends of different fibre sizes and types. Being able to understand the performance of these models in application to fibre blends will enable users to characterise these types of fibrous media more precisely. From this work, it was concluded that the Miki model (Miki, 1990) is the most accurate model to invert the airflow resistivity from acoustical surface impedance of a wide range of nonwoven blends.

1. Introduction

Airflow resistivity is an important parameter when considering the acoustic performance of a material. There have been several studies looking into this area since the original work conducted by Nichols [1], which proposed a relation between this parameter and the fibre diameter and density of a material. The realisation that airflow resistivity is directly linked to the acoustic properties of fibrous media allows for the measurement of both fibre diameter and material density rapidly and non-invasively from acoustical data, such as the surface impedance or absorption coefficient [2,3]. One question this paper addresses is how well a model can invert these parameters from a standard acoustic impedance tube test [4] performed on a nonwoven fibrous blend specimen? Another question this paper addresses is how accurate are some models which relate the fibre diameter, density and flow resistivity. To answer the first question, this paper aims to study the performance of two models used to predict the acoustical properties of fibrous media and compare the inversion results against measured and predicted flow resistivity data. To answer the second question, this paper studies the performance of three popular models which predict the flow resistivity from the material microstructure and density data. The understanding of the accuracy of these models is useful to develop new efficient fibrous products for a broad range of acoustic absorption applications and to appreciate their limitations when used for material parameter inversion.

In order to compare these models, a representative range of nonwoven fibrous media was provided by John Cotton Group Ltd. These media were composed of fibres of variable diameters and varied in density, thickness, porosity, and pore composition. A thermosetting binder fibre was also added to the blend, which, when heated, partially melts and so fixes the layers of fibres in place. Such variations were used to ascertain if there are any models which are more suited to certain types of fibrous media or if there is a model to be found which performs particularly well across all types of nonwoven fibrous media.

The paper is organised in the following manner. Section 2 presents the models which were used to predict the flow resistivity from fibre and density information or to invert it from acoustical data. Section 3 presents the experimental methodology which was used to measure acoustical and related non-acoustical characteristics of fibrous media. Sections 4 and 5 are the discussion and conclusions, respectively.

2. Model introduction

Three equations for the direct estimation of the airflow resistivity were chosen for the experiment reported in this work: (i) the Bies-Hansen equation [5]; (ii) Garai-Pompoli equation [6]; and the (iii)

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Kozeny-Carman equation [7].

The airflow resistivity values predicted with these equations were then compared to those deduced via two mathematical models, which are able to invert the airflow resistivity of fibrous media from their acoustical properties. These models were: (i) the Miki model [8]; and the (ii) Padé approximation model [9].

2.1. Bies-Hansen equation

The Bies-Hansen equation [5] relates a material's airflow resistivity to its fibre diameter and bulk density:

$$\sigma d^2 \rho_m^{-K_1} = K_2. \tag{1}$$

In this equation, σ is the airflow resistivity [Pa s/m^2], ρ_m is the bulk density of the fibres [kg/m^3], d is the mean fibre diameter within the sample [m], and both K_1 and K_2 are dimensionless empirical constants – which have values of 1.53 and 3.18×10^{-9} for fibre glass materials, respectively. It should be noted that the work by Bies and Hansen [5] assumes that the materials have a uniform fibre diameter, which is less than 15μm, and that there is a negligible binder fibre content in the material sample.

2.2. Garai-Pompoli equation

Upon applying the Bies-Hansen equation to polyester fibre samples Garai and Pompoli found that the airflow resistivity values were grossly underestimated [6]. They surmised that this was as a result of polyester samples having larger fibre diameters than the fibreglass samples Bies-Hansen originally modelled, and so the constants fitted in the Bies-Hansen model were not sufficiently accurate to predict the actual value of airflow resistivity.

Garai and Pompoli proposed new values of the coefficients K_1 and K_2 in Eq. (1). Garai and Pompoli refer to their equation as the "new resistivity model (NRM)", which is [6]:

$$\sigma = A\rho_m^B, \tag{2}$$

where $A = K_2 d^{-2}$ and $B = K_1$. A and B are free parameters and so can be calculated for varying sample compositions to obtain the best fit. Garai and Pompoli reported that the values of $A = 25.989$ and $B = 1.404$ provided the best fit for polyester fibres, and so those were the values used in this experiment.

Garai and Pompoli also reported that from their analysis of four different types of polyester materials the binder fibre percentage did not seem to impact the precision of their equation, and that it was not affected by surface smoothing treatments [6]. Theoretically, this means that their model should be accurate for a broad range of the samples presented in this paper, some of which feature differing binder percentages.

2.3. Kozeny-Carman equation

The Kozeny-Carman equation originates from the 1930s and was originally employed to relate the porosity of granular media, for example soils and sands, to airflow resistivity [2,3]. This equation has subsequently been applied to estimate airflow resistivity of textiles, especially polymer fibres using the following relationship [7]:

$$\sigma = \frac{180\mu(1-\phi)^2}{d^2\phi^3} \tag{3}$$

μ is the dynamic viscosity, which is a constant derived from Poiseuille's equation of laminar flow for a liquid, and was assigned the value of 1.81×10^{-5} Pa s for this experiment, d is the particle size, which was assumed to be equivalent to the fibre diameter in Eq. (1), σ is the airflow resistivity and ϕ is the porosity, which was calculated from the ratio of bulk material density, ρ_m, to fibre density, ρ_f, via the following equation:

$$\phi = 1-\frac{\rho_m}{\rho_f}. \tag{4}$$

From the above three equations it can be seen that the airflow resistivity of a sample is inversely dependent upon the fibre diameter squared, but the coefficients in these equations differ.

2.4. Miki model

The Miki Model was published by Miki in 1989 [8], as an improvement to the empirical model of Delany and Bazley [10]. Miki proposed some modifications to the Delany-Bazley model to yield a model that is more accurate and causal across a broader frequency range. According to the Miki model [7] the characteristic impedance of a porous medium can be calculated more accurately from:

$$z_b(f) = R(f) + iX(f), \tag{5}$$

$$R(f) = 1 + 0.070\left(\frac{f}{\sigma}\right)^{-0.632} \tag{6}$$

$$X(f) = 0.107\left(\frac{f}{\sigma}\right)^{-0.632}. \tag{7}$$

The wavenumber for sound propagation in porous media was also modified and given by the following equations:

$$k_b(f) = \alpha(f)i + \beta(f) \tag{8}$$

$$\alpha(f) = \frac{2\pi f}{c_0}\left[0.160\left(\frac{f}{\sigma}\right)^{-0.618}\right] \tag{9}$$

$$\beta(f) = \frac{2\pi f}{c_0}\left[1 + 0.109\left(\frac{f}{\sigma}\right)^{-0.618}\right]. \tag{10}$$

In the above equations f is the frequency of the sound wave (Hz), and c_0 is the speed of sound in air (m/s) and $i = \sqrt{-1}$.

2.5. Padé approximation model (PadéNUP)

The Padé approximation model was proposed by Horoshenkov et al. [9] and it makes use of the Padé approximant theory to approximate the viscosity correction function in the expressions for the characteristic impedance and wavenumber in a porous medium with non-uniform pores:

$$z_b(\omega) = \sqrt{\widetilde{\rho}_b(\omega)/\widetilde{C}_b(\omega)} \text{ and } k_b(\omega) = \omega\sqrt{\widetilde{\rho}_b(\omega)\widetilde{C}_b(\omega)}, \tag{11}$$

where $\widetilde{C}_b(\omega) = 1/\widetilde{K}(\omega)$ is the bulk complex compressibility of air in the material pores, $\widetilde{K}(\omega)$ is the dynamic bulk modulus of the air in the material pores and $\widetilde{\rho}_b(\omega)$ is the dynamic density of the air in the material pores and $\omega = 2\pi f$ is the circular frequency.

The Padé approximation model makes use of approximations for the dynamic density (Eq. (12)), and one for approximating the complex compressibility (Eq. (14)). According to this model the dynamic density can be expressed as:

$$\widetilde{\rho}_x(\epsilon_\rho) = 1 + \epsilon_\rho^{-2}\, \widetilde{F}_\rho(\epsilon_p), \tag{12}$$

where the viscosity correction function is given by a Padé approximant:

$$\widetilde{F}_\rho(\epsilon_p) = \frac{1 + \theta_{\rho,3}\, \epsilon_\rho + \theta_{\rho,1}\, \epsilon_\rho}{1 + \theta_{\rho,3}\, \epsilon_\rho}, \tag{13}$$

with $\epsilon_\rho = \sqrt{-i\omega\rho_0/\sigma_x}$. In the above Padé approximation, the coefficients $\theta_{\rho,1} = 1/3$, $\theta_{\rho,2} = \sqrt{1/2}\, e^{1/2(\sigma_s \log 2)^2}$, $\theta_{\rho,3} = \theta_{\rho,1}/\theta_{\rho,2}$ are real and positive numbers.

Similarly, the complex compressibility is: