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Periodic boundary based FFT-FISTA for sound source identification

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ABSTRACT

Compared with the conventional beamforming, the Fourier-based fast iterative shrinkage thresholding algorithm (FFT-FISTA) can effectively improve the spatial resolution and suppress the sidelobe. To furtherly achieve higher computational efficiency and better sound source identification performance, an alternative periodic boundary is utilized to replace the zero boundary of Fourier transform, a periodic boundary based FFT-FISTA is proposed in this paper. And its superiority is demonstrated by the simulation and validation experiment of equal and unequal intensity sources.

1. Introduction

For the advantages of fast measuring speed, high calculation efficiency and being suitable for middle and long distance measurement, beamforming, an array-based measurement technology, has become a common tool used to identify sound sources for aviation aircraft [1–4], express train [5], wind turbine [6], automobile [7] and so on over the last decades. However, conventional beamforming suffers poor spatial resolution at the low frequency and massive ghost images at the high frequency [8–11]. To overcome these issues, Lylloff et al. [12] put forward the Fourier based fast iterative shrinkage thresholding algorithm (FFT-FISTA) whose Fourier transform process is based on the zero boundary and compared with the Fourier-based non-negative least squares algorithm (FFT-NNLS) method. The results show that the FFT-FISTA has a higher computational efficiency and better identification performance.

To furtherly improve the zero boundary based FFT-FISTA computational efficiency and sound source identification quality, a modified Fourier-based fast iterative shrinkage thresholding algorithm based on periodic boundary is proposed in the paper. The sound source identification performance of the periodic boundary based FFT-FISTA is compared with that of zero boundary based through the simulation and experiment results.

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2. Principle of periodic boundary based FFT-FISTA

The cross-spectral imaging function is a common algorithm for near field beamforming, the resulting imaging function [13,14] is

$$B(\mathbf{r}) = \frac{1}{M}|b(\mathbf{r})| = \frac{1}{M}\frac{|\boldsymbol{v}^{\mathrm{T}}(\mathbf{r})\boldsymbol{C}\boldsymbol{v}^{*}(\mathbf{r})|}{\sqrt{\boldsymbol{w}^{\mathrm{T}}(\mathbf{r})1\boldsymbol{w}^{*}(\mathbf{r})}} \tag{1}$$

$\boldsymbol{C}$ is the cross-spectral matrix of the sound pressure signals, 1 is a unity matrix with all elements equal to 1, $\boldsymbol{v}(\mathbf{r})$ is the steering column vector and $\boldsymbol{w}(\mathbf{r}) \equiv [|v_1(\mathbf{r})|^2, |v_2(\mathbf{r})|^2, \dots |v_m(\mathbf{r})|^2 \dots |v_M(\mathbf{r})|^2]^T$, $m = 1, 2, 3, \dots, M$ is the serial number of the microphones.

$$v_m(\mathbf{r}) = \exp(-\mathrm{i}k|\mathbf{r} - \mathbf{r}_m|)/|\mathbf{r} - \mathbf{r}_m| \tag{2}$$

k is the wave number, f is the frequency and c is the propagation speed of sound, $\mathrm{i} = \sqrt{-1}$.The map is assumed to be solely generated by a set of uncorrelated point sources in the source plane. So,

$$\boldsymbol{C} = \sum_{\mathbf{r}'}\boldsymbol{C}(\mathbf{r}') = \sum_{\mathbf{r}}^{\prime} q(\mathbf{r}')|\mathbf{r}'|[\boldsymbol{v}^{*}(\mathbf{r}')\boldsymbol{v}^{\mathrm{T}}(\mathbf{r}')] \tag{3}$$

where $\mathbf{r}'$ is the position of the assumed point source and $q(\mathbf{r}')$ is its sound pressure contribution at the center of the array. Then,

$$\begin{cases} b(\mathbf{r}) = \sum_{\mathbf{r}'} q(\mathbf{r}')psf(\mathbf{r}|\mathbf{r}') \\ psf(\mathbf{r}|\mathbf{r}') = |\mathbf{r}'|\frac{\boldsymbol{v}^{\mathrm{T}}(\mathbf{r})[\boldsymbol{v}^{*}(\mathbf{r}')\boldsymbol{v}^{\mathrm{T}}(\mathbf{r}')]\boldsymbol{v}^{*}(\mathbf{r})}{\sqrt{\boldsymbol{w}^{\mathrm{T}}(\mathbf{r})1\boldsymbol{w}^{*}(\mathbf{r})}} \end{cases} \tag{4}$$

where $psf(\boldsymbol{r}|\boldsymbol{r}')$ is the PSF, The error function φ is constructed between PSF, sound source distribution and output of conventional beamforming,

$$\varphi = \frac{1}{2}\|\boldsymbol{A}\boldsymbol{q} - \boldsymbol{b}\|_2^2 \tag{5}$$

where $\|\|_2$ denotes the 2 norm, $\boldsymbol{A} = [psf(\boldsymbol{r}|\boldsymbol{r}')]$ is the known PSF matrix, $\boldsymbol{q} = [q(\boldsymbol{r}')]$ is the unknown source distribution column vector and each element is not less than zero, $\boldsymbol{b} = [b(\boldsymbol{r})]$ is the known imaging function column vector. In addition, a fast projected gradient descent algorithm is used to obtain the source pressure contribution $\boldsymbol{q}$. The expression is given by

$$\boldsymbol{q}_{l+1} = P_+\left(\boldsymbol{q}_l - \frac{1}{L}\boldsymbol{A}^{\mathrm{T}}(\boldsymbol{A}\boldsymbol{q}_l - \boldsymbol{b})\right) \tag{6}$$

where P_+ is the Euclidean projection of $\boldsymbol{q}$ onto the non-negative quadrant, $\boldsymbol{q}_l$ represents the source pressure contribution after l iterations, L is the Lipschitz constant and equal to the largest eigenvalue of $\boldsymbol{A}^{\mathrm{T}}\boldsymbol{A}$.To improve the efficiency of above-mentioned zero boundary based FFT-FISTA, the periodic boundary condition is used to replace the zero boundary of Fourier transform. The PSF matrix $\boldsymbol{A}$ is a Block Circulant with Circulant Blocks matrix under the assumption that the PSF is shift invariant. For BCCB matrix is normal, $\boldsymbol{A}^{\mathrm{H}}\boldsymbol{A} = \boldsymbol{A}\boldsymbol{A}^{\mathrm{H}}$, $\boldsymbol{A}$ has a unitary spectral decomposition, $\boldsymbol{A} = \boldsymbol{F}^{\mathrm{H}}\boldsymbol{\Lambda}\boldsymbol{F}$, where $\boldsymbol{F}$ is the two-dimensional unitary discrete Fourier transform (DFT) matrix. $\boldsymbol{F}^{\mathrm{H}} = \boldsymbol{F}^{-1}$, $\boldsymbol{F}^{-1}$ is the inverse matrix of $\boldsymbol{F}$, $\boldsymbol{\Lambda}$ is the eigenvalue matrix of PSF matrix $\boldsymbol{A}$.

$$\boldsymbol{A}^{\mathrm{T}}\boldsymbol{A} = \boldsymbol{A}^{\mathrm{H}}\boldsymbol{A} = (\boldsymbol{F}^{\mathrm{H}}\boldsymbol{\Lambda}\boldsymbol{F})^{\mathrm{H}}(\boldsymbol{F}^{\mathrm{H}}\boldsymbol{\Lambda}\boldsymbol{F}) = \boldsymbol{F}^{\mathrm{H}}\boldsymbol{\Lambda}^{\mathrm{H}}\boldsymbol{F}\boldsymbol{F}^{\mathrm{H}}\boldsymbol{\Lambda}\boldsymbol{F} = \boldsymbol{F}^{\mathrm{H}}\Delta\boldsymbol{F} \tag{7}$$

where Δ is the eigenvalue matrix of $\boldsymbol{A}^{\mathrm{T}}\boldsymbol{A}$ and equals to $\boldsymbol{\Lambda}^{\mathrm{H}}\boldsymbol{\Lambda}$.This $\boldsymbol{F}$ matrix has a very convenient property, that is

$$\sqrt{N}\boldsymbol{F}\boldsymbol{q}_l \leftrightarrow F(\boldsymbol{Q}_l), \frac{1}{\sqrt{N}}\boldsymbol{F}^{\mathrm{H}}\boldsymbol{q}_l \leftrightarrow F^{-1}(\boldsymbol{Q}_l), \boldsymbol{F}^{\mathrm{H}}\boldsymbol{F}\boldsymbol{q}_l \leftrightarrow F^{-1}(F(\boldsymbol{Q}_l)) \tag{8}$$

"F" and "F^{-1}" are Fourier transform and inverse Fourier transform, respectively, that,

$$\boldsymbol{A}\boldsymbol{q}_l = \boldsymbol{F}^{\mathrm{H}}\boldsymbol{\Lambda}\boldsymbol{F}\boldsymbol{q}_l \leftrightarrow F^{-1}(F(\boldsymbol{Q}_l) \circ F(\widetilde{\boldsymbol{A}})) \tag{9}$$

"$\circ$" represents the Hadamard product operation, $\boldsymbol{Q}_l$ is a N_r rows and N_c columns matrix reshaped from the vector $\boldsymbol{q}_l$, $\widetilde{\boldsymbol{A}}$ is reshaped from the first column of the matrix $\boldsymbol{A}$. Under the periodic boundary condition, the Fourier transform can be expressed as

$$\boldsymbol{A}^{\mathrm{T}}(\boldsymbol{A}\boldsymbol{q}_l - \boldsymbol{b}) \leftrightarrow F^{-1}[(F(\widetilde{\boldsymbol{A}}))^{\mathrm{H}} \circ F[F^{-1}[F(\boldsymbol{Q}_l) \circ F(\widetilde{\boldsymbol{A}})] - \boldsymbol{B}]] \tag{10}$$

where $\boldsymbol{B}$ is a $N_r \times N_c$ matrix reshaped from the vector $\boldsymbol{b}$. "$(\cdot)^{\circ(\cdot)}$" represents the Hadamard exponentiation.

Based on the above Fourier transform, the periodic boundary based FFT-FISTA converts the non-negative least squares problem into a Fourier-based minimization equation:

$$\varphi = \frac{1}{2}\|F^{-1}\left[F(\boldsymbol{Q}_l) \circ F(\widetilde{\boldsymbol{A}})\right] - \boldsymbol{B}\|_{\mathrm{Fro}}^2 \tag{11}$$

where $\|\|_2$ denotes the Frobenius norm. The iteration search in the negative gradient direction is applied to obtain the $\boldsymbol{Q}$, given a start matrix $\boldsymbol{Q}_0 = 0$, set $\boldsymbol{Y}_1 = \boldsymbol{Q}_0$, $t_1 = 1$, and repeat for $l \geqslant 1$ until the stopping criterion is satisfied.

① $\boldsymbol{Q}_l =_+ (\boldsymbol{Y}_l - \frac{1}{L}F^{-1}[(F(\widetilde{\boldsymbol{A}}))^{\mathrm{H}} \circ F[F^{-1}[F(\boldsymbol{Q}_l) \circ F(\widetilde{\boldsymbol{A}})] - \boldsymbol{B}]])$

② $t_{l+1} = \frac{1}{2}(1 + \sqrt{1 + 4t_l^2})$

③ $\boldsymbol{Y}_{l+1} = \boldsymbol{Q}_l + \frac{t_l - 1}{t_{l+1}}(\boldsymbol{Q}_l - \boldsymbol{Q}_{l-1})$

3. Simulation

Simulations of the monopole sources are conducted based on the cross-spectral imaging function beamforming theory. The Brüel & Kjær sector wheel microphone array with 0.65 m diameter, consisting of 36 microphones arranged in a pseudo-random pattern, is considered. The scene of interest is built to cover an area of size $1\ \mathrm{m} \times 1\ \mathrm{m}$ with 51×51 scanning points, which are arranged evenly with the spacing of 0.02 m. Assuming the monopole sources are located at $(-0.2,\ 0.2,\ 1)$ m and $(0.2,\ 0.2,\ 1)$ m in the sound source plane. Based on the free-field Green function, the cross-spectral matrix of the array microphone signals is obtained. Then, the acoustic maps are achieved by scanning each point in the focusing plane backwardly based on the cross-spectra imaging function with auto-spectra exclusion beamforming method. Finally, FFT-FISTA is used to clean the acoustic maps obtained in previous step. The display dynamic ranges are all set as 15 dB in this paper.

The acoustic maps at 5000 Hz of the conventional beamforming, zero boundary based and periodic boundary based FFT-FISTA with 1000 iterations are presented in Fig. 1(*a*–*c*), respectively. As shown in the Fig. 1(*a*), sound sources are accurately located at $(-0.2,\ 0.2,\ 1)$ m and $(0.2,\ 0.2,\ 1)$ m. Fig. 1(*b*) and (*c*) show that the deconvolution methods can effectively narrow the mainlobe, improve the spatial resolution and clear the sidelobe. Additionally, the zero boundary based and periodic boundary based FFT-FISTA can obtain accurate amplitude of sound source after 1000 iterations. Compared with the results of zero boundary based FFT-FISTA in Fig. 1(*b*), the periodic boundary based utilized in Fig. 1(*c*) can further reduce the width of the mainlobe, eliminate the sidelobe and improve the convergence.

Fig. 2 shows the results of two unequal intensity sound sources with a magnitude difference of 6 dB. The frequency and iteration times are same as previous. Under 15 dB display dynamic ranges, all the conventional beamforming and deconvolution methods can accurately identify the strong and weak source. Compared to the result of conventional beamforming in Fig. 2 (a), results of zero boundary based and periodic boundary based FFT-FISTA in Fig. 2 (b) and (c) have narrower mainlobe and less sidelobe. What is

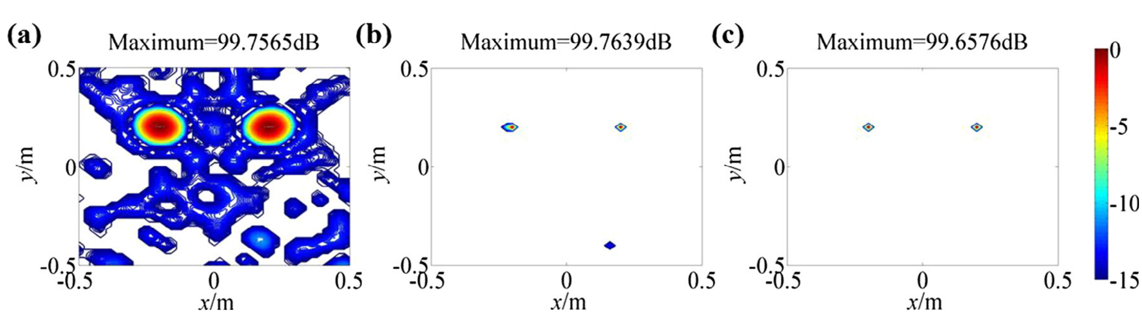

Fig. 1. Simulation acoustic maps of equal intensity sound source at 5000 Hz. (a) conventional beamforming, (b) zero boundary based FFT-FISTA with 1000 iterations, (c) periodic boundary based FFT-FISTA with 1000 iterations.