Applied Acoustics 130 (2018) 204–209

Contents lists available at ScienceDirect

Applied Acoustics

journal homepage: www.elsevier.com/locate/apacoust

Technical note

Acoustic performance of different Helmholtz resonator array configurations

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ARTICLE INFO

Keywords:
Helmholtz resonator
Transmission loss
Energy storage capacity

ABSTRACT

This paper presents a theoretical study of the acoustic performance of different Helmholtz resonator (HR) array configurations. A dual HR consisting of two HRs connected in series (neck-cavity-neck-cavity) could be considered as a serial HR array. Two HRs mounted on the same cross-section of the duct constitute a parallel HR array. A lined HR array is composed of two HRs installed on the longitudinal direction of the duct. Since HR is reactive silencer without energy consumption, the energy storage capacity (C_{TL}) of HR arrays could be defined as the area under transmission loss curve. The transfer matrix method is developed to conduct the investigation. The predicted theoretical results fit well with the Finite Element Method (FEM) simulation results. The results indicate that the installation methods have significant effects on transmission loss curves. However, the C_{TL} of the dual HR equals the C_{TL} of each single component HR mounted on the duct in despite of the dual HR having two HRs. The C_{TL} of the parallel HR array is equivalent to the C_{TL} of the lined HR array, which is twice the C_{TL} of the dual HR. The C_{TL} should therefore be considered as one of the main acoustic characteristics of a HR or an array and be taken into consideration in noise control optimization and HR design.

1. Introduction

An important application of a Helmholtz resonator (HR) that consists of a cavity communicating with an external duct through an orifice is the reduction of noise propagation in ducts. The resonance frequency of a HR is only determined by its geometries. It is therefore straightforward to design a HR with a desired resonance frequency. Owing to the characteristics of simplicity, adjustability and durability, the applications of the HR become an important area of study and have been utilized in numerous duct-structure systems, such as ventilation and air conditioning system in buildings, automotive duct systems and aero-engines, for the attenuation of noise produced by in-ducted elements [1–5].

Many researchers and engineers around the world have devoted their attention to the investigation of the HR. A lot of achievements have been made and are documented in numerous pieces of literature. Mainly through the labours of Helmholtz, Rayleigh, Ingard, Sondhauss and Wertheim, the classical lumped approach for a HR is supposed to be analogous to the mechanical mass-spring system with end-correction factors for the sake of the accuracy [6]. Furthermore, a considerable number of researchers have developed the wave propagation in both the duct and the HR in theoretical analysis from an initial one-dimensional wave propagation approach to a multidimensional approach in order to account for nonplanar effects [7–9]. Since the HR is qualified as narrow band silencer and it is only effective at its resonance peak. Various modification forms of HRs have been studied in order to improve the acoustic performance of a HR. Chanaud [10] investigated the effects of different orifice shapes and cavity geometries on the resonance frequency of HR. Tang and Sirignano [7] showed that resonance frequency of a HR was reduced by increasing the neck length. In order to lengthen the neck, an extended neck and a spiral neck were proposed by Selamet and Lee [11] and Shi and Mak [12] respectively. Cai et al. [13] compared the acoustic performance of HRs with these two types of necks. Tang [14] examined the HR with tapered necks of increasing cross-sectional area towards cavity both experimentally and theoretically. Griffin et al. [15] demonstrated the mechanically coupled HRs through a thin membrane to obtain three resonance frequencies instead of two. Xu et al. [16] derived expressions of two resonance frequencies and the transmission loss of a dual HR formed by a pair of neck and cavity connected in series.

However, there were no research literature regarding the HR's energy storage capacity until the recent work of Cai and Mak [17]. They proposed the concept of energy storage based on the transmission loss index. This paper focuses on evaluating the acoustic performance of different HR array configurations, especially the energy storage capacity of different HR arrays. Three different HR arrays are investigated here: (1) a dual HR consists of two HRs connected in series (neck-cavity-neck-cavity) could be considered as a serial HR array; (2) two HRs mounted on the same cross-section of the duct constitute a parallel HR array; (3) a lined HR array is composed of two HRs installed on the

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http://dx.doi.org/10.1016/j.apacoust.2017.09.026
Received 25 August 2017; Received in revised form 22 September 2017; Accepted 27 September 2017

longitudinal direction of the duct. As low frequencies are the main concerns in this study, the frequency range considered here is well below the duct's cutoff frequency. Hence, only planar wave is assumed to propagate through the duct. The theoretical predictions are validated by Finite Element Method (FEM) simulation. It is hoped that the analysis of energy storage capacity could contribute to the noise control optimization and the HR design.

2. Theoretical analysis of different Helmholtz resonator arrays

The sound fields inside a HR are clearly multidimensional because of sudden discontinuities area [8]. However, on the basis of low frequency range of interest in this paper, the dimensions of HRs considered here are significant small compared to the wavelengths. It is therefore that the evanescent high-order modes can be considered by introducing an end correction factor to improve the accuracy of the classical lumped approach. The effect of viscous dissipation through the necks will be ignored for simplicity.

2.1. A single Helmholtz resonator

Although a multidimensional approach provides a more accurate measure of the acoustics impedance of a HR, the main purpose here is to reveal the energy storage capacity of a HR. For this reason, the classical lumped approach is adopted here and the evanescent high-order modes are considered by introducing an end correction factor to improve the accuracy. It is therefore that the acoustic impedance of a HR is given as:

$$Z_r = j\frac{\rho_0 l_n'}{S_n\omega}(\omega^2-\omega_0^2) \tag{1}$$

where ρ_0 is air density, l_n' and S_n are the neck's effective length and area respectively, $\omega_0 = c_0\sqrt{S_n/l_n'V_c}$ (V_c is the cavity volume and c_0 is the speed of sound in the air) and ω are the resonant circular frequency and circular frequency respectively.

Once the acoustic impedance is obtained, the transmission loss of a side-branch HR mounted on a duct with cross-sectional area S_d could be expressed as:

$$TL = 20\log_{10}\left(\frac{1}{2}\left|2+\frac{\rho_0 c_0}{S_d}\frac{1}{Z_r}\right|\right) \tag{2}$$

2.2. A dual Helmholtz resonator

A dual HR formed by two HRs connected in series (neck-cavity-neck-cavity), which could be considered as a serial HR array, leads to two resonance frequencies. A dual HR could be analogous to a two degrees of freedom mechanical system, as illustrated in Fig. 2. According to Hooke's law, it should be noted that the first string has different stiffness (K_{11} and K_{12}) to the front and rear masses connected on it. By applying the Newton's second law of motion to the first mass M_1 and the second M_2 respectively yield:

$$M_1\frac{d^2x_1}{dt^2}+R_1\frac{dx_1}{dt}+K_{11}x_1 = S_1 p_0 e^{j\omega t},\quad M_2\frac{d^2x_2}{dt^2}+R_2\frac{dx_2}{dt}+K_{22}x_2 = K_{12}x_2 \tag{3}$$

where $M_1 = \rho_0 S_1 l_{n1}'$ and $M_2 = \rho_0 S_2 l_{n2}'$ are the corresponding mass of air in the necks including the end-correction factor, R_1 and R_2 are damping coefficients of necks, K_{11} and K_{12} represent the stiffness of the first spring to the first mass and second mass respectively, $K_{22} = \rho_0 c_0^2 S_2^2/V_2$ is the stiffness of the second spring, $e^{j\omega t}$ is the time-harmonic disturbance. Appling the Hooke's law to the mechanical analogy of a dual HR, the stiffness K_{11} and K_{12} could be obtained as:

$$K_{11} = \frac{\rho_0 c_0^2 S_1}{V_1 x_1}(S_1x_1-S_2x_2),\quad K_{12} = \frac{\rho_0 c_0^2 S_2}{V_1 x_2}(S_1x_1-S_2x_2) \tag{4}$$

With the introduction of Eq. (4), Eq. (3) could be rewritten as:

$$\begin{cases} M_1\frac{d^2x_1}{dt^2}+R_1\frac{dx_1}{dt}+\frac{\rho_0c_0^2S_1}{V_1}(S_1x_1-S_2x_2) = S_1p_0e^{j\omega t} \\ M_2\frac{d^2x_2}{dt^2}+R_2\frac{dx_2}{dt}+\frac{\rho_0c_0^2S_2^2}{V_1}x_2-\frac{\rho_0c_0^2S_2}{V_1}(S_1x_1-S_2x_2) = 0 \end{cases} \tag{5}$$

Substituting $x_1 = X_1e^{j\omega t}$ and $x_2 = X_2e^{j\omega t}$ into Eq. (5) and rearranging in the matrix form as:

$$\begin{bmatrix} -\omega^2M_1+j\omega R_1+\frac{\rho_0c_0^2S_1^2}{V_1} & -\frac{\rho_0c_0^2S_1S_2}{V_1} \\ -\frac{\rho_0c_0^2S_1S_2}{V_1} & -\omega^2M_2+j\omega R_2+\rho_0c_0^2S_2^2\frac{(V_1+V_2)}{V_1V_2} \end{bmatrix}\begin{Bmatrix} X_1e^{j\omega t} \\ X_2e^{j\omega t} \end{Bmatrix} = \begin{Bmatrix} S_1p_0e^{j\omega t} \\ 0 \end{Bmatrix} \tag{6}$$

where X_1 and X_2 are the magnitude of the first and the second neck's displacement respectively. Eq. (6) could be simplified as:

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}\begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = \begin{Bmatrix} p_0S_1 \\ 0 \end{Bmatrix} \tag{7}$$

where

$$\begin{bmatrix} -\omega^2M_1+j\omega R_1+\frac{\rho_0c_0^2S_1^2}{V_1} & -\frac{\rho_0c_0^2S_1S_2}{V_1} \\ -\frac{\rho_0c_0^2S_1S_2}{V_1} & -\omega^2M_2+j\omega R_2+\rho_0c_0^2S_2^2\frac{(V_1+V_2)}{V_1V_2} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}.$$

According to Eq. (7), the relation of X_1 and p_0S_1 could be described as $X_1 = p_0S_1a_{22}/(a_{11}a_{22}-a_{12}a_{21})$. It is therefore that the acoustic impedance of the dual HR could be obtained as:

$$Z_r = \frac{p_0}{j\omega X_1S_1} = \frac{1}{j\omega S_1^2}\frac{a_{11}a_{22}-a_{12}a_{21}}{a_{22}} \tag{8}$$

By ignored the effect of viscous dissipation through the necks for simplicity ($R_1 = R_2 = 0$), Eq. (8) could be rewritten as:

$$Z_r = \frac{1}{j\omega S_1^2}\frac{M_1M_2\omega^4-\rho_0c_0^2\left[M_1S_2^2\left(\frac{1}{V_1}+\frac{1}{V_2}\right)+M_2S_1^2\frac{1}{V_1}\right]\omega^2+\frac{\rho_0^2c_0^4S_1^2S_2^2}{V_1V_2}}{\rho_0c_0^2S_2^2\left(\frac{1}{V_1}+\frac{1}{V_2}\right)-M_2\omega^2} \tag{9}$$

It is therefore that the transmission loss of the dual HR could be obtained through Eq. (2).

2.3. A parallel Helmholtz resonator array

Two Helmholtz resonators mounted on the same cross-section of the duct is illustrated in Fig. 2. The acoustic impedance of these two HRs can be calculated by Eq. (1), expressed as Z_{r1} and Z_{r2} respectively. The continuity conditions of sound pressure and volume velocity at the duct-neck interface give:

$$p_1 = p_2 = p_{f1} = p_{f2},\quad S_du_1 = S_du_2+\frac{p_{f1}}{Z_{r1}}+\frac{p_{f2}}{Z_{r2}} \tag{10}$$

where p with subscript represents sound pressure, u_1 and u_2 are the particle velocity at point 1 and point 2 respectively.

The relation between point 1 and to point 2 could be obtained by combining the continuity conditions as:

$$\begin{bmatrix} p_1 \\ \rho_0c_0u_1 \end{bmatrix} = \begin{pmatrix} 1 & 0 \\ \frac{\rho_0c_0}{S_d}\frac{Z_{r1}+Z_{r2}}{Z_{r1}Z_{r2}} & 1 \end{pmatrix}\begin{bmatrix} p_2 \\ \rho_0c_0u_2 \end{bmatrix} \tag{11}$$

Then the transmission loss of the parallel HR array can be determined by the four-pole parameters method as [18]:

$$TL = 20\log_{10}\left(\frac{1}{2}\left|2+\frac{\rho_0c_0}{S_d}\frac{Z_{r1}+Z_{r2}}{Z_{r1}Z_{r2}}\right|\right) \tag{12}$$