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The deterministic bispectrum of coupled harmonic random signals and its application to rotor faults diagnosis considering noise immunity

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ABSTRACT

This paper considers the properties of a bispectrum estimate when applied to a system with quadratic nonlinearity excited by the superposition of harmonics in the presence of additive Gaussian noise. This is compared using signal-to-noise ratios, to the power spectrum. Numerical examples were included to verify the results. In addition, an application of the use of the bispectrum to detect rotor faults in rotating machinery through detection of quadratic phase coupling is presented. The paper aims to clarify the use of the bispectrum to detect non-linearity in time series and presents the background theory on the bispectrum and in its application. Further, we show how patterns in the bispectrum are useful for identifying the frequency (or *bifrequency*) components involved in the non-linear interaction. The properties of interest are insensitivity to Gaussian noise and it's ability to detect quadratic phase coupling. The study aims to expand the domain of induction machines faults diagnosis. To verify the theoretical development, stator currents which were collected from a test bed based on a 18.5 kW three-phase squirrel cage induction machine have been used in a steady-state condition.

1. Introduction

Over the past decades, higher order spectra (HOS) [1], also called as polyspectra, have established a status as a sophisticated mathematical and signal processing tool for nonlinear system analysis.

Traditional power spectrum, which is defined as the Fourier transform (FT) of the autocorrelation sequence (the second-order cumulant), does not give any information about the phase of system frequency response and is unable to give any evidence of nonlinearity for a system. However, the HOS [2] is defined as the multidimensional FT of higher order cumulants of a stationary random process and can overcome the inability of power spectra to detect these nonlinearities.

From the structure of HOS, it is possible to deduce various properties of signals that do not appear when using the power spectrum. For example, many different signals can have the same correlation function or the same power spectrum, but they can be distinguished by using HOS. Furthermore, there are various methods of signal processing using HOS that can solve problems that cannot be addressed using only second order statistics.

Often there are situations in which the interaction between two harmonic components causes a contribution to the power at their sum and/or difference frequencies. For example an important class of nonlinear interaction, named quadratic phase coupling (QPC), involves frequency triplet, F_0, F_1 and F_2. QPC means that the sum of the phases at $F_0(\theta_0)$ and $F_1(\theta_1)$ is the phase at frequency $F_0 + F_1$ (i.e. $\theta_0 + \theta_1$), which is often an indication of second-order nonlinearities. In certain applications, it is necessary to determine if peaks at harmonically related positions in the power spectrum are, in fact, phase coupled. Since the power spectrum suppresses all phase relations, it cannot provide the answer. Bispectral analysis is a powerful tool to detect QPC and has been applied successfully to evaluate QPC types of nonlinear effects [1,2]. This is best illustrated in the example given in Figs. 2–7.

The bispectrum is the third order spectrum and it results in a frequency-frequency-amplitude relationship which shows coupling effects between signals at different frequencies [1,2]. Therefore, the bispectrum [3] is sensitive to the non-Gaussianity of signals and can effectively extract information due to deviations from Gaussianity. If a signal is Gaussian, then its bispectrum would be identically zero: for a non-Gaussian signal, the bispectrum can be non-zero [1,2].

As a result, the bispectrum has been widely used in a number of detection applications including biomedical engineering [4], plasma physics [6], engineering structures [7], mechanical systems

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[8,9], the coupling assessment between modes in a power generation system [10], and broken bars diagnosis in induction machines [11–15], etc.

There are very few references [16–19] dedicating to the bispectrum for nonlinear system outputs. This is because deriving the bispectrum for nonlinear system outputs is much more difficult as compared to linear systems. Without any additional specification, only general structure can be used in this case, as the nonlinearity does not introduce any precise definition of the statistics of a signal. Therefore, if we require explicit analytic expressions of HOS, it is necessary to introduce statistical models of signals that can represent physical phenomena and have a structure leading to possible explicit calculations. This is one of the main purposes of this paper [19,20].

The paper is organized as follows. In the next section, the specific definition of the higher HOS, namely the power spectrum and the bispectrum is provided, and the corresponding frequency domain symmetries are also discussed. Section 3 introduces the concept of coupled signals by examples. Section 4, the shows the application of bispectrum to detect nonlinearities in quadratic systems, in presence of additive Gaussian noise. Section 5, will be dedicated to the analysis of experimental results before giving some conclusions and future works.

2. General higher-order spectral analysis

2.1. Power spectrum

The physical interpretation of the classical power spectrum is a one-dimensional function of frequency and has demonstrated very powerfully in modelling linear phenomena. The discrete power spectrum is defined as,

$$P_{xx}(f) = E\{X(f)X^{*}(f)\} = E\{|X(f)|^{2}\} \tag{1}$$

where X^{*} denotes the complex conjugate of X, $X(f)$ is the discrete FT of $x(n)$ and E is the expectation operator a zero-mean stationary random signal, given by:

$$X(f) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi fn}{N}}. \tag{2}$$

where N is the number of samples present in the signal.

The power spectrum in Eq. (1) is defined regardless of whether the signal is zero-mean. It can also be defined in a short time form for non-stationary signals if the discrete-version over finite records as in Eq. (2) is used. Assumptions about stationarity are only made when applied to random signals.

Because all phase information is destroyed in computing the power spectrum, the power spectrum is incapable of detecting phase coupling signatures.

2.2. Bispectrum

The next higher-order spectrum is the bispectrum, a two-dimensional function of frequency, which is very powerful in detecting and quantifying quadratic effects in a time-series [1,2].

Meanwhile, the bispectrum gives a description of the statistical relationships between the signal frequency components. These relationships are, in particular, the nonlinear indicator in the signal.

The bispectrum is defined as,

$$B(f_1, f_2) = E\{X(f_1)X(f_2)X^{*}(f_1 + f_2)\}. \tag{3}$$

The bispectrum is all about statistics (higher order statistics). It is the FT of the third cumulant or moment. Nonlinearity has an effect on this cumulant that is captured by the bispectrum. The expectation operation is very important in this context and cannot be ignored especially in the detection and quantification of phase coupling. It involves 'ensemble averaging' for an estimate such that if phases are random, the bispectrum goes to zero and if phases are coupled it does not.

It can be noted that the bispectrum presents twelve symmetry regions [1,2]. Hence, the analysis can take into consideration only a single non-redundant region. Hereafter, $B(f_1, f_2)$ will denote the bispectrum in the triangular region $\Im$ shown in Fig. 1 and defined by:

$$\Im = \{(f_1, f_2) : 0 \leqslant f_2 \leqslant f_1 \leqslant f_e/2, f_1 + f_2 \leqslant f_e/2\}.$$

where f_e is the sampling frequency. Regions of computation are discussed in Refs. [1,2].

For the bispectrum to be non-zero at (f_1, f_2), the FTs at f_1, f_2, and $f_1 + f_2$ must be non-zero.

As a result, the bispectrum can be used to solve a number of practical problems effectively. Examples are expressed as follows [1–6]:

- Gaussian processes: If $x(n)$ is a stationary zero-mean Gaussian process, its bispectrum $B(f_1, f_2)$ is identically zero.
- Linear phase shifts: While the power spectrum suppresses all phase information, the bispectrum does not.
- Non-Gaussian white noise: It's bispectrum is flat, only if the third order autocorrelation is an impulse at the origin [26].
- The bispectrum is quantified measure of HOS. It is the Fourier transform of the third cumulant or moment. Nonlinearity has an effect on these cumulant that is captured by the bispectrum.

For the interested reader, a theoretical introduction of HOS is given in [1,2].

2.3. Estimation

In general, the expected values coming from Eqs. (1) and (3) need to be estimated from a finite quantity of available data.

Non-parametric (conventional) approaches to bispectrum estimation may be either direct or indirect. The usual, indirect method requires estimation of the third-order cumulant and computation of the 2D-FT. Instead, we employ the direct method, which may be achieved by dividing a signal into M overlapping segments with k as a subscript, $k = 1, 2, 3, \ldots, M$. A windowing function has been applied to each segment and the FTs of all segments are averaged [1]. The aim is to reduce the variance of the estimate by increasing the number of records M [1,2].

The estimated bispectrum $\hat{B}(f_1, f_2)$ is given by:

$$\hat{B}(f_1, f_2) = \frac{1}{M}\sum_{k=1}^{M} X_k(f_1)X_k(f_2)X_k^{*}(f_1 + f_2) \approx E\{X(f_1)X(f_2)X^{*}(f_1 + f_2)\}. \tag{4}$$

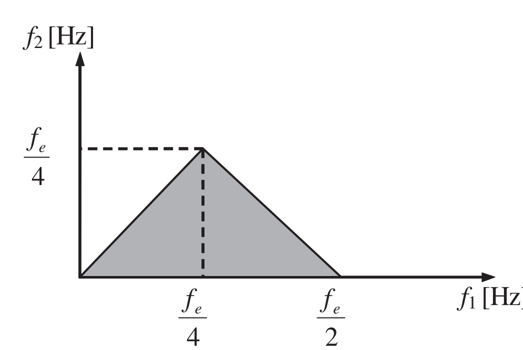

Fig. 1. A non-redundant region of the bispectrum.