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String/frets contacts in the electric bass sound: Simulations and experiments

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ABSTRACT

For particular playing techniques such as "pop" or "slap" in the electric bass guitar, the string collides with frets, producing a percussive sound used in different music styles. The string/frets contacts introduce a nonlinearity which is investigated both numerically and experimentally in this paper. A physical model, based on a modal description of the string, is implemented with an unconditionally stable scheme. Simulations including a string/structure coupling and the two polarisations of the string are confronted to controlled experiments, showing a good agreement for increasing amplitudes of initial conditions. A parametric study is then conducted numerically in order to highlight the influence of physical parameters on the transient behaviour and raises questions related to tuning and playing issues.

1. Introduction

The solid-body electric bass has a recent history, opening up during the first half of the 20th century [1]. Originally designed to increase the sound level and to be played with better precision than the double bass, the solid-body electric bass was inspired by the solid-body electric guitar with four heavy strings tuned to the same notes as the double bass [1]. The sound of the instrument is a result of an electro-acoustic chain beginning with the string vibratory motion. This latter is then of prime importance and can be disrupted by its coupling with the structure of the instrument. The string vibration decay can vary depending on the finger position, due to the induced boundary condition, and dead spots can be produced at a fingering position. This phenomenon may be explained through a linear description of the coupling between the neck and the string [2,3]. However, possible nonlinear features are not investigated in these studies. In particular, among playing techniques adopted by musicians, some rely on a percussive aspect of the sound, implying contacts between vibrating strings and the neck. Two typical such playing modes are "pop", for which the string is plucked hardly enough to generate contact, and "slap", for which the string is hit with the thumb, also resulting in string/neck contacts [4]. The string/obstacle contact introduces nonlinearity, that has been widely studied numerically. The highly nonlinear behaviour of the string vibrating in presence of an obstacle makes the problem stiff and implies numerical difficulties, in particular regarding stability. Among existing numerical methods, some models use waveguides [5–7], which reproduce effects through signal processing, or energy-based methods [8–11], ensuring a good stability to employed schemes. Some models rely on a modal description of the string [12,13,10,11], which possibly enables a fine description of the string linear characteristics such as damping. Only a few studies present experimental signals with an isolated string or a complete instrument in order to give a comparison point for their simulations [12,14,11,15], the latter being applied to slap on electric basses. In [16], a listening test is performed to evaluate the synthesis algorithm.

The present paper aims at presenting a numerical tool to simulate musical strings vibrating against a unilateral distributed obstacle, and confronting it to experiments into detail. The method is applied to the pop attack on electric basses, for which the string is plucked with a sufficiently large amplitude so that contact occurs and gives the sound a percussive timbre during the attack transient. The objective of such a numerical tool is to move forward the comprehension of the string behaviour when colliding with a fretboard, through the study of some key parameters. The employed numerical model is presented in Section 2. A controlled experimental protocol is then proposed in Section 3. Numerical and experimental signals are confronted in Section 4, and a

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numerical parametric study is led in order to highlight the influence of some parameters on the resulting sound, some of which may be related to playing and instrument making issues.

2. Model

2.1. Model of a string vibrating against an obstacle

A stiff string of length L, mass per unit length μ, tension T, Young's modulus E and moment of inertia I is considered. The string (see Fig. 1) vibrates in the presence of an obstacle having a profile $g(x)$ which is constant along (Oy) and under the string at rest. The transverse displacements $u(x,t)$ and $v(x,t)$ of the string along (Oz) and (Oy) respectively are described by Eq. (1), in which the subscript t (respectively x) refers to a partial derivative with respect to time (respectively space):

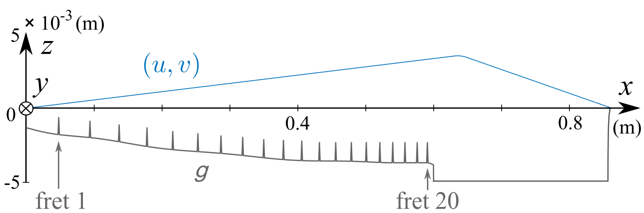

Fig. 1. A string of length L vibrating against a bass guitar fretboard represented by the function g.

$$\mu u_{tt} - T u_{xx} + EI u_{xxxx} = f \tag{1a}$$
$$\mu v_{tt} - T v_{xx} + EI v_{xxxx} = f_f, \tag{1b}$$

where the right-hand sides f (contact force per unit length) and f_f (friction force per unit length) are fully described later.

Simply supported boundary conditions at the string endpoints are employed, this corresponds to a common assumption for musical strings with a weak stiffness, see *e.g.* [12,17]. For the sake of conciseness, the next equations are only detailed for the vertical displacement u, but of course also apply to the horizontal displacement v. Boundary conditions read, $\forall t \in \mathbb{R}^+$:

$$u(0,t) = u(L,t) = u_{xx}(0,t) = u_{xx}(L,t) = 0. \tag{2}$$

The displacement is then spatially discretised by using N_m eigenmodes:

$$u(x,t) = \sum_{j=1}^{N_m} q_j(t)\phi_j(x), \tag{3}$$

with $\phi_j(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{j\pi x}{L}\right)$ and q_j the jth modal amplitude.

Inserting this expression in Eq. (1a), using standard Galerkin projection technique and adding losses, one finally obtains a system of oscillators for the unknown $\mathbf{q} = [q_1, q_2, \dots q_{N_m}]^T$ gathering the modal amplitudes as:

$$\mu(\ddot{\mathbf{q}} + \mathbf{\Omega}^2\mathbf{q} + 2\mathbf{\Upsilon}\dot{\mathbf{q}}) = \mathbf{F}, \tag{4}$$

where $\mathbf{\Omega}$ and $\mathbf{\Upsilon}$ are diagonal matrices with coefficients $\Omega_{jj} = \omega_j = 2\pi\nu_j$, ν_j being the jth eigenfrequency, and $\Upsilon_{jj} = \sigma_j$, which corresponds to the jth damping coefficient.

A penalty approach is selected to express the contact force per unit length, following [9,11]:

$$f(\eta(x,t)) = K[\eta(x,t)]_+^{\alpha}, \tag{5}$$

where $\eta(x,t) = g(x) - u(x,t)$ represents the penetration of the string into the barrier, and $[\eta]_+ = \frac{1}{2}(\eta + |\eta|)$. The regularised contact force thus depends on two parameters K and α, and derives from a potential ψ:

$$f = \frac{d\psi}{d\eta}, \quad \text{where } \psi(\eta) = \frac{K}{\alpha+1}[\eta]_+^{\alpha+1}. \tag{6}$$

The friction force per unit length f_f, acting on the polarisation along (Oy), is selected as a regularised empirical Tresca friction law [18,19], and reads:

$$f_f(u, v_t) = A\begin{cases} 1 & \text{if } v_t < -s \text{ and } u < g \\ -v_t/s & \text{if } |v_t| \leqslant s \text{ and } u < g \\ -1 & \text{if } v_t > s \text{ and } u < g \\ 0 & \text{if } u \geqslant g, \end{cases} \tag{7}$$

where v_t is the transverse velocity of the string along (Oy), and A (N.m^{-1}), $s > 0$ (m.s^{-1}) are two *ad hoc* parameters. A number of studies in the literature use a regularised friction force [20,21]. Note that such a friction force only allows one equilibrium position, this may lead to incorrect behaviours in some configurations [22].

In order to take into account the vibrations of the neck, the mobility at the nut is then added to complete the model. In the case of solid-body electric guitars and basses, it has been shown that the bridge mobility is negligible as compared to that at the nut [3,2]. Moreover, as detailed in Section 2.2, the coupling is weak so that taking into account the nut mobility only alters linear characteristics.

2.2. Linear characteristics models

The influence of the dispersion due to the stiffness of the string, though small, needs to be taken into account. Also, under the previously exposed assumption of weak coupling at the nut and as done in [3] for electric guitars, the eigenfrequencies are modeled following the relationship, for each mode j:

$$\nu_j = j\frac{c}{2L}\left(1 + \frac{Bj^2}{2} + \frac{\mu c}{j\pi}\mathrm{Im}(Y_{nut}(\omega_{0,j}))\right), \tag{8}$$

where $c = \sqrt{\frac{T}{\mu}}$ is the wave velocity of the ideal string, $B = \frac{\pi^2 EI}{TL^2}$ is the inharmonicity coefficient and Y_{nut} is the mobility at the nut, evaluated at $\omega_{0,j} = j\frac{\pi c}{L}$. In the present study, B is deduced from measurements (see Section 3.2).

The modal loss factor is evaluated thanks to the model exposed in [12,3]. For mode j, the quality factor Q_j is used to express the modal damping factor σ_j *via* $Q_j = \pi\nu_j/\sigma_j$, where Q_j is modeled as:

$$Q_j^{-1} = Q_{j,\mathrm{air}}^{-1} + Q_{j,\mathrm{ve}}^{-1} + Q_{\mathrm{te}}^{-1} + \frac{\mu c^2}{\pi L \nu_j}\mathrm{Re}(Y_{nut}(\omega_j)). \tag{9}$$

In this model, the subscripts air, ve and te respectively refer to losses due to air friction, viscoelastic and thermoelastic effects.

Contribution of friction with air writes:

$$Q_{j,\mathrm{air}}^{-1} = \frac{jc}{2L\nu_j}\frac{R}{2\pi\mu\nu_j}, \tag{10}$$

where $R = 2\pi\eta_{\mathrm{air}} + 2\pi d_{eq}\sqrt{\pi\eta_{\mathrm{air}}\rho_{\mathrm{air}}\nu_j}$, with η_{air} and ρ_{air} the dynamic viscosity coefficient and the air density respectively. Usual values (for standard temperature and pressure conditions) are selected here as: $\eta_{\mathrm{air}} = 1.8 \times 10^{-5}$ kg·m^{-1}·s^{-1} and $\rho_{\mathrm{air}} = 1.2$ kg·m^{-3}. Viscoelastic effects are supposed to be concentrated in the string core [12], so that their contribution to global losses is given by:

$$Q_{j,\mathrm{ve}}^{-1} = \frac{4\pi^2\mu E_{core} I_{core}\delta_{\mathrm{ve}}}{T^2}\frac{\nu_{0,j}^3}{\nu_j}, \tag{11}$$

where E_{core} is the Young's modulus of the core, $I_{core} = \pi r_{core}^4/4$ is the moment of inertia of the core, with r_{core} the core radius, and $\nu_{0,j} = \frac{jc}{2L}$