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A monotonic two-step iterative shrinkage/thresholding algorithm for sound source identification based on equivalent source method

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ABSTRACT

Near-field acoustical holography (NAH) based on equivalent source method (ESM) is an efficient technique applied for sound source identification. However, the conventional ESM with ℓ_2 norm regularization cannot produce a satisfactory solution in high frequency, where the average array inter-element spacing is larger than half a wavelength. Therefore, conventional ESM with Tikhonov regularization is restricted to relatively low frequency. To overcome the issue, an alternative method called monotonic two-step iterative shrinkage/thresholding algorithm for near-field acoustical holography is proposed. In the algorithm, another existing algorithm called Wideband Acoustical Holography (WBH) is used to generate the threshold, and also be used as a benchmark for comparison. Simulated measurements based on an irregular microphone array are performed to compare the performances of the Tikhonov regularization, WBH and MTwIST. The results suggested that the proposed method can identify the sound sources more accurately than Tikhonov regularization in full frequency range, and it performs better than WBH in the case of identifying coherent sources at relatively low frequency. The experiments demonstrated the validity and the practicability of the proposed method.

1. Introduction

Near-field acoustic holography (NAH) is an advanced technique for identifying sound source and for visualizing sound field. NAH based on two-dimensional spatial Discrete Fourier transform (DFT) is first proposed by Williams [1,2] in 1980s. As the evanescent wave component is obtained in the near field measurement, DFT-based NAH can produce high resolution images of sound sources regardless of the size of the wavelength. The DFT-based NAH, in spite of its efficiency, has been limited to source surface (as well as holograms) with simple geometries, for example, planar and cylindrical surfaces. This requirement has not always been met in many industrial applications where irregularly shaped sound sources have been presented. Under these circumstances, the Boundary Element Method (BEM) [3] has been proposed for irregularly shaped sources in which a series of constant element is used for approximating the field. However, BEM requires the evaluation of the Neumann Green's function, which is tedious because of the integrations, its applications are limited. As an alternative approach to BEM, wave superposition method (WSM) [4,5], also named equivalent source method (ESM) [8,9], was proposed by Koopmann in 1989. According to the theory of ESM, the sound field radiated by an arbitrarily shaped source can be substituted by a series of virtual equivalent sources on a fictitious surface. An equation concerning the equivalent source strength and the measured sound pressure is established. Then the equivalent sources strength vector can be solved by equating the sound pressure measured on the holographic surface to the sound pressure generated by these virtual equivalent sources. With the calculated equivalent source strength vector, the sound pressure exterior to the radiator can be evaluated, according to each concerned frequency. Since the ESM has the properties of patch nearfield acoustical holography (Patch NAH) [6–8], it can be used to perform local measurement for sources in large size. Besides, ESM also has some other particular advantages such as robust calculation and fast speed. For these reasons, this approach has been widely studied.

In the application of ESM, the transfer matrix between equivalent sources and the measurement positions is always underdetermined because the number of microphones is limited in most acoustic studies. To handle the underdetermined inverse problem in conventional ESM, ℓ_2 norm regularization method such as Tikhonov [9,10] is typically employed to stabilize the solution process and find the optimal equivalent source amplitude. However, conventional ESM with Tikhonov regularization focuses on relatively

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low frequency, and the identification accuracy in middle to high frequencies remains a limitation open to investigation.

In recent years, Compressive Sensing technology based on sparse regularization has received more and more attention in the field of signal/image processing [11,12]. A sparse sequence means that the number of nonzero components is quite small compared with its dimension. Under some conditions, it is possible to reconstruct the sparse signals with significantly fewer measurements (i.e., microphones) than classically required. As most components are zero, the equivalent source strength vector can be considered sparse in ESM-based NAH.

In a sparsity framework, the sparse set of equivalent source amplitude is calculated by solving an objective function containing a ℓ_1 norm penalty for equivalent source strength vector. For example, Chardon et al. [13] applied sparsity and Compressive Sensing principles to near-field acoustic holography. Fewer microphones are required compared with Tikhonov regularization when doing holography calculation on a vibrating plate. Suzuki [14] published a similar method, but a monopole/dipole point source model of the same type as ESM is used, and the sparsity is enforced by use of ℓ_1 norm penalty on the solution vector. The proposed method is called Generalized Inverse Beamforming (GIB), focusing on rather long measurement distances. Chardon et al. used Matlab convex optimization software to solve the inverse problems including ℓ_1 norm penalty, while Suzuki developed a specific iterative solver. More recently, Jorgen Hald [15] proposed Wideband Acoustical Holography (WBH), where a residual quadratic function is defined first, then the sparse solution is solved with a two-step method, including a steepest decent iteration and a thresholding process in each iteration. However, WBH is recommended to identify the coherent sound sources above 0.7 times the frequency of half wavelength.

The objective of this paper is to propose an alternative ESM-based algorithm that can identify the sources more accurately. Parallel with conventional ESM, a model of ESM-based NAH is conducted first. Different from the conventional ESM with Tikhonov regularization, the equivalent source strength vector is solved using sparse regularization under the principle of Compressive Sensing. In the process of seeking the optimal solution, the monotonic two-step iterative shrinkage/thresholding (MTwIST) is employed, which is a monotonic version of the two-step iterative shrinkage/thresholding (TwIST) algorithm. TwIST was originally proposed in the field of image processing and used for image restoration [16–18]. As an effective iterative algorithm, the MTwIST has recently been used for impact reconstruction, and achieved good results [19]. Inspired by the good performances in image processing and impact force reconstruction, the MTwIST is modified to solve the sound source identification problem based on ESM, particularly for coherent sound sources in middle to high frequency.

The present paper describes an application of the MTwIST in the field of ESM-based sound source identification. In Section 2, the basic theory of ESM-based NAH is described first. After that, conventional Tikhonov regularization method and WBH are briefly introduced. Then, MTwIST is modified to approximate the optimal solution of the objective function. In Section 3, numerical simulations including single-source, coherent-source are conducted to compare the performances of Tikhonov regularization, WBH and MTwIST. In Section 4, the practicability of MTwIST is verified by identifying the given sound sources. Finally, several conclusions are drawn in Section 5.

2. Algorithm

2.1. Brief description of equivalent source method

According to the theory of ESM, the sound pressure radiated by an arbitrary-shaped source can be substituted by a series of virtual sources located on a surface interior to the body of the radiator. The amplitudes of these equivalent sources are determined by applying boundary conditions on the measurement surface. The virtual equivalent source surface can be placed on any surfaces. But for simplifying the calculation, the equivalent source surface is set to be a rectangular plane near the real sources as shown in Fig. 1.

Assume that there are N virtual equivalent sources on equivalent source surface and M microphones set up on the measurement surface. The sound pressure at the m-th measurement point can be expressed as

$$p(m) = \sum_{n=1}^{N} g(\mathbf{r}_m|\mathbf{r}_n)q_n \tag{1}$$

here,$g(r_m|r_n)$is the monopole pressure-pressure transfer function linking n-th equivalent source to m-th microphone, and it can be expressed as in [20]

$$g(\mathbf{r}_m|\mathbf{r}_n) = \frac{\exp(-jk\|\overrightarrow{r_{mn}}\|)}{4\pi\|\overrightarrow{r_{mn}}\|} \tag{2}$$

where $k = \omega/c$ is the wave number, ω is the angular frequency, $\|\overrightarrow{r_{mn}}\|$ is the distance between the considered source-microphone couple. Then, the relationship between measured sound pressure and equivalent source strength can be described in matrix vector notation as:

$$\mathbf{p} = \mathbf{Aq} \tag{3}$$

where $\mathbf{A}$ (size $M \times N$) is the pressure transfer matrix between the equivalent sources and the measurement points, $\mathbf{p}$ (size $M \times 1$) is the complex pressure vector measured by the microphone array, $\mathbf{q}$ (size $N \times 1$) is the complex amplitude of these equivalent sources.

In the application of ESM-based NAH, the transfer matrix $\mathbf{A}$ is always underdetermined as the number of microphones is limited in most acoustic studies. To handle the ill-conditioned problem encountered in the solution procedure of vector $\mathbf{q}$, regularization methods are typically implemented to stabilize the minimization process.

After calculating the optimal equivalent source strength vecto, the sound pressure on any reconstruction surface can be obtained in the matrix form as:

$$\mathbf{p}_R = \mathbf{Hq} \tag{4}$$

with $\mathbf{H}$ the transfer matrix between the equivalent sources and the reconstruction points. The core issue in ESM-based NAH is to com-

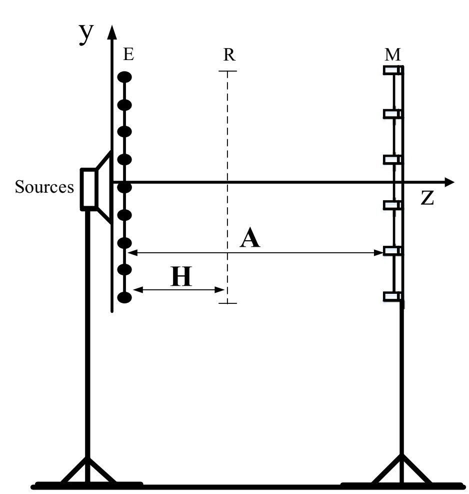

Fig. 1. Geometry of measurement surface M, equivalent source surface E and reconstruction surface R.