Applied Acoustics 121 (2017) 1–13

Contents lists available at ScienceDirect

Applied Acoustics

journal homepage: www.elsevier.com/locate/apacoust

Band gap property analysis of periodic plate structures under general boundary conditions using spectral-dynamic stiffness method

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ARTICLE INFO

Article history:
Received 21 May 2016
Received in revised form 16 January 2017
Accepted 19 January 2017
Available online 3 February 2017

Keywords:
Spectral dynamic stiffness method
Vibration band gap
Periodic plate structure
General boundary conditions

ABSTRACT

Vibration band gap properties of periodic rectangular plate structures with general boundary conditions are studied using the spectral-dynamic stiffness method (S-DSM). Material and dimensional properties are assumed to vary periodically in the whole plate structure. In the S-DSM, the general solution governing each unit plate is firstly derived with the aid of spectral method and then the dynamic stiffness matrix is established by substituting the general solution into corresponding boundary equations. Finally, the global spectral dynamic matrix for the whole periodic plate structure is assembled in a similar way to the finite element method (FEM). The harmonic responses are calculated to illustrate the band gap properties of the periodic structures. By comparing the results obtained by the FEM, it can be verified that the S-DSM is of superior accuracy and convergence rate. A parametric study is also conducted to investigate the effects of the material properties, geometrical parameters and structural damping on the vibration band gap behaviors of the periodic structures.

1. Introduction

The propagation of elastic waves in periodic systems has received much attention over the years [1–8]. More recently, there has been an active area of research in a special type of inhomogeneous materials and geometries (the so-called phononic crystals) whose elastic coefficients or geometrical parameters vary periodically in space. Many remarkable mechanic properties of such structures have been discovered and investigated. Among them, the band gap property in frequency domain is one of the most distinctive properties which is caused by the elastic waves reflected back and forth between the interfaces of the adjacent cells when they propagate in the periodic structures. In the band gap frequency range, nearly no elastic wave can propagate through the periodic structure. The vibration band gap properties enable the periodic structure to be widely used in engineering fields, such as, frequency filters, noise controls, and vibrationless environments for high-precision mechanical systems [4,12,13].

A variety of both theoretical and numerical studies have been conducted to investigate the wave filtering behavior in periodic structures. For example, Yan and Wang [9–11] developed a wavelet-based method (WBM) to study the elastic band gaps of phononic crystals. The eigen-equation for the phononic crystals was derived by expanding the displacement fields in the wavelet basis functions. Based on the transfer matrix method (TMM), Ruzzene and Baz [14] and Marino and Berardo [15] studied the dynamic behaviors of the periodic rods with shape memory inserts which can act as the filters to weaken the elastic wave propagation along the rods. Yu et al. [16] extended the TMM to calculate the band gaps of flexural waves in the Euler-Bernoulli beam and studied its transverse vibration band gap behaviors. To investigate the flexural vibration band gap properties of the periodic beams, Xiang and Shi [12] derived the theoretical equations of beam by employing the Bloch-Floquet theorem and then solved the equations using the differential quadrature method. Olhoff et al. [17] analyzed a repeated inner segment of the optimized beams using the waveguide finite element method (WFEM) to study the band-gap for travelling waves. The authors aimed to maximize the band gaps between two adjacent natural frequencies and to study the associated creation of periodicity of the optimized beam designs. Hussein et al. [18] designed the target frequency band structures characterizing longitudinal wave motion of one-dimensional periodic unit cell by using a multi-objective genetic algorithm. The design approach provided practical guides for forming bounded structures with advanced vibration, shock isolation and filtering characteristics. Considerable vibration studies on varies of periodic cylindrical shell structures are also conducted in recent years [19–21]. Shen et al. [22] and Wen et al. [23] took the advantages of band gap

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http://dx.doi.org/10.1016/j.apacoust.2017.01.024

properties of periodic composite pipes to provide a novel avenue for vibration control and noise reduction in complex piping systems. Sorokin and Ershova [24] employed the Floquet theory and boundary integral equations to study the steady-state free vibration of non-uniform elastic cylindrical shells with and without internal heavy fluid loading. Shen et al. [25,26] illustrated the mechanism of wave propagation and vibration transmission in functionally graded material periodic cylindrical shell and studied its capability to control sound and vibration.

The structures of periodically placed plates are commonly encountered in various practical engineering. Much effort has been devoted to investigating the dynamic characteristic of such periodic plate structures in the past decades. In the early years, Sigalas and Economou [27] calculated the elastic waves propagation in plates with solid inclusions placed periodically in the host plate. Langley and Smith [28] used the statistical energy analysis (SEA) to study the effects of disorder in the stiffener spacing on the high frequency vibration transmission through a periodic plate system. In recent years, Wang et al. [29] developed an improved FEM to study the physical mechanisms for the formation of periodically stiffened plate structures with any number or orientation of stiffeners. Wu et al. [30,31] applied the spectral element method (SEM) to investigate the dynamic behavior of periodic plate structures with classic plate theory (CPT) and Mindlin plate theory (MPT). But both the improved FEM and SEM have their limitations. For the FEM, the number of element should be large enough to obtain credible results in high-frequency range. In addition, computational efficiency and memory space will be unacceptable if the element number becomes very large. Although the SEM is of excellent accuracy and efficiency, it is only suitable for limited boundary conditions for plate structures, i.e., at least two opposite simply supported boundaries. Recently, the dynamic stiffness method (DSM) has drawn more and more attention due to its high computational efficiency and accuracy compared with the FEM resulting from the exact shape functions and lower degrees of freedom. And it has been developed for a bunch of one-dimensional (1D) elements such as rods and beams [32–34] and two dimensional (2D) rectangular plates using various plate theories [35,36]. However, for plate structures, the traditional DSM is only available for the cases where at least a pair of opposite boundaries are simply supported, i.e. Levy plates. More recently, Liu and Banerjee [37] developed a novel S-DSM by introducing the idea of general spectral method, which had removed the boundary constraints of the traditional DSM. This idea certainly opened up a new way to solve vibration problems of plate structures in an accurate and efficient manner. Afterwards, Banerjee and his co-workers extended and extended the S-DSM to study the free vibration of composite plate structures and plate assemblies with arbitrary non-uniform elastic supports mass attachments and elastic coupling constraints [38–40].

In the present work, the S-DSM is employed to calculate the forced vibration responses of periodic plate structures under general boundary conditions and investigate the band gap behaviors in frequency domain. The S-DSM is more efficient than the FEM and can cope with more complex boundary conditions than the SEM. This paper is arranged as follow: in Section 2, the formulation of the dynamic stiffness matrix of the periodic plate structure is introduced briefly, after which the applications of external load and arbitrary prescribed boundary conditions are given. In Section 3, firstly, several numerical examples are given to validate the accuracy and convergence of the current results. Then, a parametric study is conducted to investigate the band gap behaviors of the periodic palate structures under the effects of material and dimensional properties. Finally, the conclusions of this paper are given in Section 4.

2. Mathematical formulation

The periodic plate structure shown in Fig. 1 is considered in this work. The system consists of N repetitions of alternating plates A and B. Plates A and B are made of different material properties or geometric dimensions. A set of parallel edges are under general boundary conditions and the other set of edges are free or coupled with the edges of the adjacent plates.

2.1. Governing differential equation

Fig. 2 shows a rectangular Kirchhoff plate in its local coordinate system. The length, width and thickness of the plate element are $2a$, $2b$ and h, respectively. In this study, the plate is assumed to be homogeneous, isotropic, elastic and of uniform thickness. The thickness of the plate is much smaller than the other two dimensions, and the inertia of shear and rotation are supposed to be negligible. Then, based on the Kirchhoff thin plate theory, the differential equation can be described as [41].

$$D\left(\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2\partial y^2}+\frac{\partial^4 w}{\partial y^4}\right)+\rho h\left(\frac{\partial^2 w}{\partial t^2}\right)=0 \quad (1)$$

where $D=Eh^3/[12(1-\nu^2)]$ is the bending stiffness, E is the Young's modulus, ν and ρ are the Poisson's ratio and density of the plate respectively.

By introducing the harmonic representation, the transverse displacement can be given as

$$w(x,y,t)=W(x,y)e^{i\omega t} \quad (2)$$

where ω is the angular frequency, $W(x, y)$ is the amplitude of the transverse displacement in frequency domain. By substituting Eq. (2) into Eq. (1), one can get

$$D\left(\frac{\partial^4 W}{\partial x^4}+2\frac{\partial^4 W}{\partial x^2\partial y^2}+\frac{\partial^4 W}{\partial y^4}\right)-\rho h\omega^2 W=0 \quad (3)$$

Based on the thin plate assumptions, the rotations of the transverse normal, bending moments and transverse shear forces can be written as

$$\varphi_x=-\frac{\partial W}{\partial x}, \quad (4)$$

$$\varphi_y=-\frac{\partial W}{\partial y}, \quad (5)$$

$$M_x=-D\left(\frac{\partial^2 W}{\partial x^2}+\nu\frac{\partial^2 W}{\partial y^2}\right), \quad (6)$$

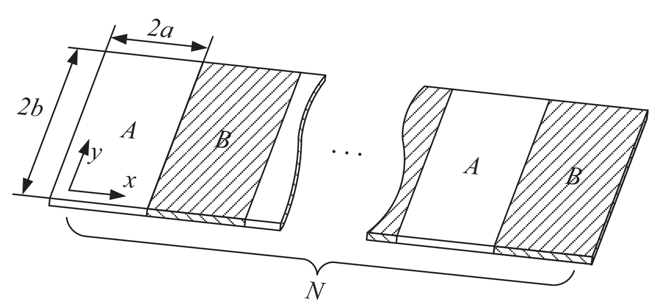

Fig. 1. Arrangement of the periodic plate structure.