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Wave based method for vibration analysis of elastically coupled annular plate and cylindrical shell structures

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ABSTRACT

Free and forced vibrations of elastically coupled thin annular plate and cylindrical shell structures under elastic boundary conditions are studied through wave based method. The method is involved in dividing the coupled structure into shell segments and annular plates. Flügge shell theory and thin plate theory are utilized to describe motion equations of segments and plates, respectively. Regardless of boundary and continuity conditions, displacements of individual members are expressed as different forms of wave functions, rather than polynomials or trigonometric functions. With the aid of artificial springs, continuity conditions between segments and plates are readily obtained and corresponding governing equation can be established by assembling these continuity conditions. To test accuracy of present method, vibration results of some coupled structures subjected to different boundary and coupling conditions are firstly examined. As expected, results of present method are in excellent agreement with the ones in literature and calculated by finite element method (FEM). Moreover, effects of annular plates, elastic coupling and boundary conditions, excitation and damping are also studied. Results show that normal displacement of annular plate mainly affects free vibrations of the coupled structures, while tangential displacement has the greatest effect on forced vibrations as meridinoal or normal excitation forced on the annular plate.

1. Introduction

Plates and shells are widely used in engineering and vibration characteristics of these independent members have been deeply investigated by lots of researchers. Most of these investigations were well summarized by Leissa [1] and Qatu [2,3]. On the other hand, due to the complexity involved in modeling and solution process, literature about coupled annular plate and cylindrical shell structures is rare, whereas the coupled structures also play an importance role in industrial vessels, missiles, submarines and so forth. Since vibrations of the coupled structures have significant effect on their performance, knowing vibration behaviors of the coupled structures is important in analysis and design process. Of course, FEM is a powerful tool and commercial FEM programs, such as ANSYS, ABAQUS and NASTRAN, have been well developed. Nevertheless, FEM has inherent disadvantages in efficiency. To identify the mode shape of a certain frequency, the solution must be extracted for each mode shape and classified one by one, which is a time-consuming and tiresome work. In addition, to meet requirements of convergence, the number of elements rapidly increases as the range of analysis frequency increases, which seriously reduces computation efficiency and increases storage space in return. To this end, proposing an accurate and efficient method to evaluate vibrations of coupled plate-shell structures is considerable.

To the authors' best knowledge, there are some but not many papers studying vibrations of coupled plate-shell systems. In the early research, Smith and Haft [4] and Takahashi and Hirano [5,6] were the typical researchers investigating vibrations of coupled plate-shell systems. Neglecting in-plane motions of the circular plate, Smith and Haft [4] analytically determined natural frequencies of a cylindrical shell closed by a circular plate at one end and clamped at the other end. Takahashi and Hirano [5,6] studied vibrations of cylindrical shells with circular plates at ends and intermediate section. By using boundary and continuity conditions, the Lagrangian in terms of unknown boundary values of displacements was formulated, and corresponding frequency equations were obtained by minimizing the Lagrangian. Soon afterwards, more and more researchers began to study vibrations of the coupled structures. In those studies, the transfer matrix method [7–9], state space method [10,11], receptance method [12–14], Rayleigh-Ritz approach [15–18] and finite element method [19–21] were usually adopted.

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Irie et al. [7] used the transfer matrix method to discuss free vibrations of joined conical-cylindrical shells. Natural frequencies and mode shapes of an annular plate-cylindrical shell system were presented as the special case. Then, the same method was adopted by Yamada et al. [8] to analyze free vibrations of circular cylindrical double-shell systems closed by end plates. Liang and Chen [9] combined the vibration theory with the transfer matrix method to calculate natural frequencies and mode shapes of a conical shell with an annular or a round end plate. State space method (SSM), which is similar with the transfer matrix method, was proposed by Tavakoli and Singh [10] for the eigensolution of axisymmetric joined/hermitic thin shells. Then they adopted SSM to investigate free vibrations of a hermetic can consisting of one cylindrical shell and two end circular plates [11]. Since distinct boundaries required in SSM, a "pinhole" with free edge was additionally introduced at the center of end circular plates.

Huang and Soedel [12,13] used the receptance method for free vibrations of a simply supported cylindrical shell welded one or more circular plates at arbitrary axial positions. Then, the receptance method was extended by Yim et al. [14] to analyze vibrations of a clamped-free cylindrical shell with a plate attached to the shell at arbitrary axial position.

Rayleigh-Ritz method is another extensively adopted approach. With the help of artificial springs, Cheng and Nicolas [15] used the variational principle to establish analytic formulation for free vibration analysis of a cylindrical shell-circular plate system with elastic coupling and boundary conditions. In the analysis, circumferential and radial displacements at two ends of the shell were assigned as 0, which simplified admissible functions of cylindrical shells. Correspondingly, eigenfunctions of shear diaphragm supported shells were utilized. In addition, in-plane motions of circular plates were not considered to meet the prior boundary conditions. Yuan and Dickinson [16] applied the Rayleigh-Ritz approach for free vibrations of cylindrical shell and plate systems, which were coupled by artificial springs. General orthogonal polynomials were selected as trial functions of separate components regardless of boundary and continuity conditions. The Fourier-Ritz method, which were widely used for vibration analysis of conical shells, cylindrical shells, annular plates and so forth [22–25], was extended by Ma et al. [18] to analyze free and forced vibrations of coupled cylindrical shell and annular plate systems. In theoretical formulation, artificial springs were also utilized to restrain displacements at boundaries and to combine plates and shells. Differing from the displacement functions adopted by Yuan and Dickinson [16], the modified Fourier series composed of standard Fourier series and auxiliary functions were selected. However, only classic boundaries were taken in account in the results analysis, and annular plates with pinhole, rather than circular plates, were adopted to model circular plates.

In vibration analysis of the coupled structures, transmission of structure-borne vibrations between the coupled shell and plate is important and difficult. Tso and Hansen [26] emphatically studied transmission characteristics of vibration waves through the junction of two semi-infinite cylindrical shells and one annular plate. Motions of the cylindrical shells were described by Donnell-Mushtari equations while motions of the plate were represented by Bessel functions. The results showed that wave transmission properties of a cylindrical shell were approximately same with those of a flat plate after the cut-on of the plate longitudinal wave.

From above review it is clearly known that the cited literature mainly was focused on free vibrations of plate-shell systems with rigid coupling and classic boundary conditions. However, the weld defects or bolted connections can lead to incompletely coupling conditions. In addition, more complex boundary conditions, rather than classic ones, may appear in engineering applications. To compensate for these shortcomings, the paper presents an accurately and efficiently analytic method for both free and forced vibration analysis of elastically coupled annular plate and cylindrical shell structures with elastic boundary conditions. Wave based method, which has been adopted by the research group of the authors to study vibrations of cylindrical shells [27–29], is extended to establish the governing equation of the coupled thin structures. The method is involved in dividing the coupled structures into shell segments and annular plates. Flügge shell theory and thin plate theory are respectively utilized to describe motion equations of these thin segments and plates. Regardless of boundary and continuity conditions, displacements of individual members are expressed as different forms of wave functions. In addition, artificial springs are employed to restrain displacements at edges and to couple annular plates with the cylindrical shell at any axial locations. Since Flügge shell theory and thin plate theory are adopted, the present method is applicable to analyze linear vibrations of coupled thin cylindrical shell and annular plate structures. Correspondingly, only thin shells and plates are considered in present paper. On the other hand, in practical engineering applications, e.g. submarines, missiles, rockets and so on, the coupled structures mainly consist of thin shells and plates. In the circumstances, present method can be utilized for vibration analysis of these engineering structures. Although adopting the first shear deformation shell theory, high shear deformation shell theory or 3-D elastic theory can lead to more extensive applications, utilizing thin shell and plate theories to establish the governing equation is much more concise in theoretical derivation process and the computation efficiency is higher in some degree. Additionally, as the coupled structures consist of thin shells and plates, high accurate results can be also predicted, which can be readily observed from above cited references. Besides, Lee and Kwak [30] pointed out that Donnell-Mushtari theory was not sufficiently accurate to calculate natural frequencies of cylindrical shells while there was no discernible difference among other shell theories, including Sanders theory, Love-Timoshenko theory, Reissner theory, Flügge theory and Vlasov theory. On the whole, adopting Flügge shell theory and thin plate theory to investigate the titled problem can predict accurate results and the accuracy will be emphatically discussed in the following analysis.

2. Theory equations

Schematic diagram of a cylindrical shell with P-1 uniform annular plates is shown in Fig. 1(a). h, R and L are the thickness, radius and length of the shell. h_p and R_1 are the thickness and inner radius of annular plates. L_i is the axial length between the ith and $(i+1)$th annular plate. The global cylindrical coordinate system (r, θ, X) is also presented. In the figure, end plates are not taken into account, but the case can be easily considered.

2.1. Description of wave based method

As adopting wave based method to establish governing equation of the coupled structure, the first step is to decompose the coupled structure into separate shell segments and annular plates according to the locations of annular plates. In Fig. 1(b), local coordinate systems of a shell segment and a plate are presented. Meanwhile, positive directions of displacement and force resultants at edges are also given. After decomposition, to solve motion equations of separate substructures, appropriate wave functions, rather than polynomials or trigonometric functions used in the literature [15–25], are adopted to express displacement functions of individual substructures, which is the essence of wave based method. In addition, it should be emphasized that the wave functions can