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Technical note

Improvement of Fourier-based fast iterative shrinkage-thresholding deconvolution algorithm for acoustic source identification

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ABSTRACT

Fourier-based fast iterative shrinkage-thresholding algorithm (FFT-FISTA) is a novel beamforming deconvolution method for acoustic source identification. Regrettably, there is a deficiency of failing to identify the acoustic source far from the center of calculation plane, when utilizing Fourier-based beamforming deconvolution method with the traditional regular focus point grid. This paper devotes to overcoming the deficiency by a new irregular focus point grid. Simulations and validation experiment both show that FFT-FISTA with an irregular focus point grid can overcome the forementioned deficiency. Besides, it is superior to the FFT-FISTA with a regular focus point grid in terms of spatial resolution improvement, sidelobe removement and source quantification, when the acoustic source is close to the center of calculation plane. This work contributes to enlarge the acoustic source identification area of FFT-FISTA and improve its performance.

1. Introduction

Beamforming has become a popular technique for noise source identification in the field of aerospace, vehicle and other engineering, due to the advantages of fast measuring speed, high computational efficiency, suitability for medium-and-long distance measurement, etc. [1]. However, conventional delay and sum (DAS) algorithm suffers from poor spatial resolution, serious sidelobe contaminations [2].

DAS can be regarded as the convolution between array point spread function (PSF) and outputs of conventional beamforming. Deconvolution eliminates the PSF from the outputs of conventional beamforming to suppress sidelobes and improve spatial resolution effectively, which can demystify the acoustic source identification results [3]. After a long period of study, some classic deconvolution algorithms have been proposed, including CLEAN [4,5], Richardson-Lucy (RL) [6,7], nonnegative least-squares (NNLS) [8], and deconvolution approach for the mapping of acoustic sources (DAMAS) [9]. These early classic deconvolution algorithms involve high-dimension matrix operations, which results in numerous variables, huge time consumption and thus little practical engineering application value. In order to improve the computational efficiency, some Fourier-based algorithms are proposed, such as DAMAS2 and DAMAS3 [10], and FFT-NNLS and FFT-RL [11]. Compared with the other three methods, FFT-NNLS has the advantages of rapid convergence, high spatial resolution and powerful sidelobes removing capability [12]. However, there are deficiencies in those Fourier-based algorithms with a regular focus point grid. When the acoustic source is away from the center of calculation plane (the position of assumptive shift invariant PSF), identification results deteriorate, and acoustic source position and pressure contribution deviate much from the real values. It can be ascribed to the fact that the actual PSF is shift variant, failing to meet the precondition of FFT. Xenaki [13] proposed a new irregular focus point grid, which can improve the shift invariance of PSF.

Recently, based on fast iterative shrinkage-thresholding algorithm (FISTA) [14], Lylloff proposed Fourier-based fast iterative shrinkage-thresholding algorithm for acoustic source identification [15]. Compared with FFT-NNLS, FFT-FISTA has higher spatial resolution, faster computational efficiency and better robustness, though the previous deficiencies in those Fourier-based algorithms with a regular focus point grid. Lylloff, in the discussion section of [15], expected that utilizing an irregular focus point grid may overcome the limitation of FFT-FISTA. This paper makes efforts for that.

This paper is organized as follows: first, in Section 2, the acoustic source identification theory of conventional DAS and FFT-FISTA deconvolution are illuminated. Then, in Section 3, a new irregular focus point grid generated by the transformation of coordinates

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is introduced. In Sections 4 and 5, simulation and experimental results acquired by different algorithms are compared respectively. Finally, Section 6 gives the conclusions of the paper.

2. Basic theory

2.1. Conventional DAS and its PSF

Fig. 1 shows the layout of beamforming acoustic source identification. The symbols "•" denote microphones and $\boldsymbol{r}_m$ is the position of mth microphone. Beamforming acoustic source identification utilizes these microphones to receive acoustic signal and discretizes acoustic source calculation plane to form a focus point grid, where $\boldsymbol{r}$ is the position of one focus point. When focusing on each grid point in the reversed direction, acoustic signal is processed based on specific algorithms. It makes the outputs of focus points of real acoustic source strengthened to form "mainlobe" and the outputs of the other focus points attenuated to form "sidelobe". Thus, the acoustic source can be identified effectively. DAS is the conventional beamforming algorithm [16] and its output is:

$$b(\boldsymbol{r}) = \frac{1}{M}\frac{\boldsymbol{v}(\boldsymbol{r})\boldsymbol{C}\boldsymbol{v}^{\mathrm{H}}(\boldsymbol{r})}{\sqrt{\boldsymbol{g}(\boldsymbol{r})\boldsymbol{U}\boldsymbol{g}^{\mathrm{H}}(\boldsymbol{r})}} \tag{1}$$

where M is the number of microphones, $\boldsymbol{C}$ is the cross-spectral matrix of sound pressure signal received by array microphones, $\boldsymbol{U}$ is $M \times M$ matrix whose elements are all unit, and "$(\cdot)^{\mathrm{H}}$" denotes conjugate transpose. $\boldsymbol{v}(\boldsymbol{r}) = [v_1(\boldsymbol{r}), v_2(\boldsymbol{r}), \ldots v_m(\boldsymbol{r}), \ldots v_M(\boldsymbol{r})]$ is the steering row vector and $\boldsymbol{g}(\boldsymbol{r}) = [|v_1(\boldsymbol{r})|^2, |v_2(\boldsymbol{r})|^2, \ldots |v_m(\boldsymbol{r})|^2, \ldots |v_M(\boldsymbol{r})|^2]$. The elements of $\boldsymbol{v}(\boldsymbol{r})$ are given by

$$v_m(\boldsymbol{r}) = \frac{\mathrm{e}^{-jk|\boldsymbol{r}-\boldsymbol{r}_m|}}{|\boldsymbol{r}-\boldsymbol{r}_m|} \tag{2}$$

where $k = 2\pi f/c$ is wavenumber, f is frequency, and c is sound velocity.

Each acoustic source is assumed incoherent, thus the cross-spectral of sound pressure signals received by array microphones equals to the sum of cross-spectral generated at the array microphones by each acoustic source, which is given by

$$\boldsymbol{C} = \sum_{\boldsymbol{r}_s}\boldsymbol{C}(\boldsymbol{r}_s) = \sum_{\boldsymbol{r}_s} q(\boldsymbol{r}_s)\boldsymbol{v}^{\mathrm{H}}(\boldsymbol{r}_s)\boldsymbol{v}(\boldsymbol{r}_s) \tag{3}$$

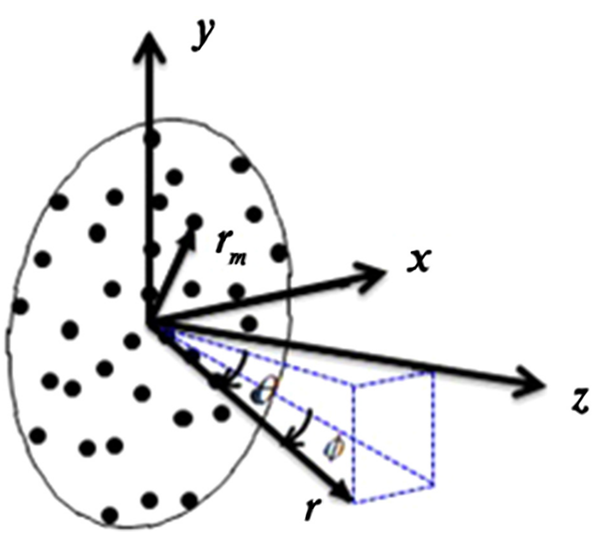

Fig. 1. Layout of beamforming measurement.

where $\boldsymbol{r}_s$ is the position of the acoustic source and $q(\boldsymbol{r}_s)$ is the source sound pressure contribution to the array center. Array PSF $psf(\boldsymbol{r}|\boldsymbol{r}_s)$ is defined as the beamforming contribution at the focus point $\boldsymbol{r}$ generated by the point source at $\boldsymbol{r}_s$ whose sound pressure contribution (sound pressure level at the array center generated by acoustic sources) is unit, namely

$$psf(\boldsymbol{r}|\boldsymbol{r}_s) = \frac{|\boldsymbol{r}_s|^2}{M}\frac{\boldsymbol{v}(\boldsymbol{r})\boldsymbol{v}^{\mathrm{H}}(\boldsymbol{r}_s)\boldsymbol{v}(\boldsymbol{r}_s)\boldsymbol{v}^{\mathrm{H}}(\boldsymbol{r})}{\sqrt{\boldsymbol{g}(\boldsymbol{r})\boldsymbol{U}\boldsymbol{g}^{\mathrm{H}}(\boldsymbol{r})}} \tag{4}$$

According to Eqs. (1), (3) and (4), the beamforming output can be written as

$$b(\boldsymbol{r}) = \sum_{\boldsymbol{r}_s} q(\boldsymbol{r}_s)psf(\boldsymbol{r}|\boldsymbol{r}_s) \tag{5}$$

2.2. FFT-FISTA deconvolution

The deconvolution algorithms establish an equation as follows:

$$\boldsymbol{b} = \boldsymbol{A}\boldsymbol{q} \tag{6}$$

where $\boldsymbol{b}$ is $N \times 1$ column vector of beamforming output, and N is the number of focus points. $\boldsymbol{A}$ is $N \times N$ matrix composed by PSF of source at each focus point and $\boldsymbol{q}$ is $N \times 1$ column vector of unknown source sound pressure contribution. The equation is a typical linear inverse problem [17]. The deconvolution algorithms can iteratively solve $\boldsymbol{q}$ and remove the influence of PSF.

In order to improve calculation efficiency, the array PSF is assumed spatial shift invariant, which means that PSF depends on the relative position between the observation point and acoustic source instead of its specific position [10]. Array PSF at the center of acoustic source calculation plane is usually used as the spatial shift invariant PSF, denoted by $psf(\boldsymbol{r}-\boldsymbol{r}_s)$. The beamforming output can be written as Eq. (7).

$$b(\boldsymbol{r}) = \sum_{\boldsymbol{r}_s} q(\boldsymbol{r}_s)psf(\boldsymbol{r}-\boldsymbol{r}_s) \tag{7}$$

Then ,

$$\boldsymbol{B} = \boldsymbol{Q} * \boldsymbol{PSF} = F^{-1}(F(\boldsymbol{Q}) \circ F(\boldsymbol{PSF})) \tag{8}$$

where $\boldsymbol{B} = [b_{ij}(\boldsymbol{r})]$ is $N_h \times N_r$ beamforming output matrix, and N_h is the column number of focus points, and N_r is the row number of focus points. $\boldsymbol{Q}$ is $N_h \times N_r$ source sound pressure contribution matrix, and $\boldsymbol{PSF} = [psf(\boldsymbol{r}-\boldsymbol{r_s})]$ is $N_h \times N_r$ matrix of the spatial shift invariant PSF. Besides, the symbol "$*$" denotes convolution, the symbol "$\circ$" denotes Hadamard product, and F and F^{-1} denote the 2D Fourier transform and its inverse. Based on Eq. (8), FFT can be adopted to improve calculating efficiency.

The sound pressure contribution is solved by means of minimizing the following equation:

$$\frac{1}{2}\|F^{-1}(F(\boldsymbol{Q}) \circ F(\boldsymbol{PSF})) - \boldsymbol{B}\|_{Fro}^2 \tag{9}$$

where $\|\cdot\|_{Fro}$ denotes Frobenius norm and the elements of $\boldsymbol{Q}$ are non-negative. FFT-FISTA is an effective solving method and its process is initializing $\boldsymbol{Q}_0 = \boldsymbol{0}$, setting $\boldsymbol{y}_1 = \boldsymbol{Q}_0$ and $t_1 = 1$, and conducting the following iterative process until the iterations l are satisfied.

(1) $\boldsymbol{Q}_l = \rho_+(\boldsymbol{y}_l - F^{-1}(F(\boldsymbol{PSF}_R) \circ F(F^{-1}(F(\boldsymbol{y}_l) \circ F(\boldsymbol{PSF})) - \boldsymbol{B}))/L)$

(2) $t_{l+1} = \frac{1}{2}\left(1 + \sqrt{1 + 4t_l^2}\right)$

(3) $\boldsymbol{y}_{l+1} = \boldsymbol{Q}_l + ((t_l - 1)/t_{l+1})(\boldsymbol{Q}_l - \boldsymbol{Q}_{l-1})$