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Mode identification of broadband Lamb wave signal with squeezed wavelet transform

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ABSTRACT

Multiple wave modes often exist in the ultrasonic guided waves simultaneously, and these modes are dispersive, so the guided wave signals are very complex, even for the relatively simple situation of a narrowband excitation. The guided wave signals are even more difficult to analyze for broadband excitations. Time-frequency representations are appropriate for the analysis of the guided wave signals considering their non-stationary and transient nature. As a post-processing tool, the squeezed wavelet transform is studied for broadband Lamb wave mode identification in this work. The influence of the parameters of the Gabor mother wavelet on the performance of the transform is analyzed in details. It is found that the product of the σ parameter of the used Gauss function and the center frequency ω_0 of the wavelet decides the overall time and frequency resolutions, so a proper selection of the value of this product $\sigma\omega_0$ is crucial for the squeezed wavelet transform. The squeezed wavelet transform is first applied to the analysis of a synthesized signal for verification. Then it's applied for mode identification of a simulated broadband Lamb wave signal. By traversing the value of $\sigma\omega_0$, a roughly optimum analysis performance is achieved for the squeezed wavelet transform for the case of $\sigma\omega_0 = 11$, where the modes are well separated and the interferences between the modes are minimal. It's proved that as an alternative tool, the squeezed wavelet transform could be used for the analysis of a broadband Lamb wave signal. An additional benefit of this transform is that it permits reconstruction of the original signal or its components, which is not possible for the reassigned scalogram.

1. Introduction

Ultrasonic waves propagating along the extension direction of bounded elastic media like the plate and the pipe are called the guided waves. Accordingly the structures they propagate in are waveguides. Guided waves are increasingly used in the inspection of various plate-like and pipe-like critical structures in different fields because they can be used to check the whole line along the propagation direction in the structure from one single point, thus reducing the time consumption unavoidable in the traditional bulk wave-based point-by-point ultrasonic inspection.

Despite the advantages, multiple modes often exist in the guided waves, and these modes are dispersive in the sense that the propagating velocities of the wave modes are functions of the frequency. Because of this dispersion phenomenon, the guided waves are more complex than the bulk waves, speaking of their propagations and interactions with the defects. Even when one approximately pure mode is generated with a narrowband excitation, the features of the guided waves might be mixed together in the time domain, thus making the interpretation of the received guided wave signal difficult. If a broadband excitation is applied, the complexity of the signal is even higher. This situation demands an effective analyzing tool for mode identification from the guided wave signals.

Guided wave signals are typical transient and non-stationary signals with time-varying frequency components. The common tool for the analysis of the non-stationary signals is the time-frequency representations (TFRs). Unlike the original unprocessed pure time domain description or the pure frequency domain description provided by the Fourier transform, the TFRs map the signal as a 2D function of both time and frequency. A by-product of the TFR is just what we're most interested in, the evolution of the frequencies of the components contained in the signal with time.

With the TFRs at our disposal and taking into account the distance of propagation of the guided waves, we can convert the theoretical group velocity dispersion curves of the guided waves

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from the frequency-velocity plane to the time-frequency plane. With this method, we can directly tell what modes are present in the received signal, so it helps greatly with the interpretation of the guided wave signals. This process was first seen in the work of Prosser et al. [1].

There're mainly two types of TFRs, i.e. the linear TFRs and the bilinear TFRs. In the linear TFRs the signal is projected to a group of time-frequency atoms [2], and the results of the TFRs are the weights of projection on these atoms. The earliest linear TFR is the short-time Fourier transform (STFT), a localized version of the Fourier transform. The STFT and the derived spectrogram are relatively easy to comprehend and are used widely. One problem of this TFR is that with the restriction imposed by the uncertainty principle, one can't obtain arbitrarily high time and frequency resolutions simultaneously. Another problem is that its time and frequency resolutions are fixed, once the window function is selected.

Another popular linear TFR is the continuous wavelet transform (WT), which is in fact a time-scale transform. Roughly speaking, the scale parameter is the reciprocal of the frequency. The time-scale atoms of the WT are generated from a mother wavelet, and the atoms form a time-scale dictionary with varying time/translation and scale parameters. At a low frequency (high scale), a longer time window (which means a narrower frequency window) is used, so the WT has lower time resolution and higher frequency resolution. At a high frequency (low scale), a shorter time window (which means a broader frequency window) is used, so the WT has higher time resolution and lower frequency resolution. The automatic adjustment of the resolutions is the main advantage of the WT. As a linear TFR, the WT is also limited by the uncertainty principle, so we can't achieve arbitrarily high time and frequency resolutions simultaneously.

Wavelets were already used in the analysis of ultrasonic guided wave signals. The Mexican Hat wavelet is a real wavelet, and it was used to measure the group velocity of the Lamb waves, so as to obtain the corrosion thickness in the plate [3]. The Mexican Hat wavelet was also used to analyze the signals generated by EMATs to identify the guided wave modes, for the tomography of artificial defects in the plate [4]. Besides the real wavelets, the complex Morlet/Gabor wavelets are often used. The Morlet wavelet was applied for the transient wave analysis in the dispersive media [5], the study of the interactions of the Lamb waves with circumferential notch in an aluminum alloyed pipeline [6], the research on the interactions of the Lamb waves with hidden corrosion defects in the aircraft aluminum structure [7], and the analysis of the multi-mode guided wave signals in the multi-wire cables [8]. Liu used the Gabor wavelet to analyze the signals of the circumferential guided waves to detect axial cracking in the pipeline [9]. Lee used the Gabor wavelet for the analysis of the signals of the guided waves propagating in the rock bolts [10].

Different from the linear TFRs, the bilinear TFRs directly correspond to the energy distributions, i.e. they map the energy of the signal as a function of time and frequency. The most important bilinear TFR is the Wigner-Ville distribution (WVD). The WVD has perfect concentration for the single component linear chirp signal, while because it has the form of a product followed by integration, cross terms emerge for the analysis of multi-component signals with the WVD. To lower the influences of these cross terms, the complex analytic signal corresponding to the original real signal is generally used in the computation of the WVD, and smoothing is introduced as in the pseudo WVD (PWVD) and the smoothed PWVD (SPWVD), although the smoothing is obtained at the price of lower time-frequency resolutions. The PWVD was used in the dispersion analysis of the Lamb waves propagating in the graphite/epoxy plates [1,11].

With the limitation imposed by the uncertainty principle, the linear TFRs have constrained time-frequency resolutions. While the bilinear TFRs, represented by the WVD, have interferences because of the cross terms. To improve the readability of these TFRs, Auger 'rediscovered' the reassigned time-frequency and time-scale representations [12]. With these reassigned versions of the original TFRs, better time-frequency concentrations are achieved. Niethammer used the reassigned spectrogram to obtain the dispersion curves of laser-generated multiple Lamb wave modes in the aluminum plate [13,14]. The reassigned spectrogram was later used in other applications like locating the defects [15].

Besides the normal TFRs, another tool or algorithm for non-stationary and nonlinear signal analysis is the empirical mode decomposition (EMD) in time domain combined with Hilbert transform (HT). The EMD method is used to decompose a signal into intrinsic mode functions (IMFs). Then HT is applied to the IMFs to obtain instantaneous frequency data. During the recent years, the EMD and HT method is increasingly used in mode recognition of Lamb waves. Zhang applied the method to analyse both directly arriving and boundary reflected Lamb wave modes of opposite types (S0 and A0) [16]. The EMD and HT method was also used to extract arrival times of Lamb waves for imaging applications [17]. These work only used narrow-banded excitations, so we will not explore further the EMD and HT method here.

In this work, an alternative wavelet post-processing technique called squeezing, proposed by Daubechies et al. [18,19], is studied for the analysis of multimode Lamb wave signal. With firstly a review of the common TFRs and the reassignment concept, the selection of the parameters of the mother wavelet is investigated, and then the squeezing theory is introduced. The squeezed wavelet transform is applied first to a synthesized signal, then to the simulated broadband Lamb wave signal. The parameters of the mother wavelet are traversed to obtain a roughly optimum performance. Although the squeezed wavelet transform provides no better performance for the analysis of broadband Lamb wave signal than the reassigned scalogram, it proves to be an alternative tool and has the additional advantage of permitting reconstruction of the original signal or its components, which is not possible for the reassigned TFRs.

2. Brief review of the time-frequency representations and the reassigned linear representations

2.1. The time-frequency representations

The basic tool for the frequency domain analysis of a signal $f(t)$ is the Fourier transform (FT),

$$F(\omega) = \int_{-\infty}^{+\infty} f(t)\mathrm{e}^{-\mathrm{j}\omega t}\,\mathrm{d}t \tag{1}$$

in which ω is the angular frequency in rad/s.

The inverse Fourier transform, or the reconstruction formula from the known Fourier transform coefficients is,

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} F(\omega)\mathrm{e}^{\mathrm{j}\omega t}\,\mathrm{d}\omega \tag{2}$$

The time information is completely lost in the Fourier transform (1), and the frequency information is absent from the original time-domain representation as in (2). The Fourier transform is only satisfactory for the representation of stationary signals whose frequency contents don't change with time. While in reality we're more often confronted with non-stationary signals with time-evolving frequency contents. One example is the recorded music signal, and all the rhythms, beautiful or not, rely on the changing frequency contents. The guided wave signal that we're interested