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# Modeling of random ground roughness effects by an effective impedance and application to time-domain methods

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ABSTRACT

Natural grounds can exhibit small scale geometric irregularities, compared to the acoustic wavelength, known as ground roughness. This roughness has a noticeable effect on sound pressure levels and produces a surface wave. In the context of prediction methods improvement for outdoor sound propagation, using an effective impedance appears to be an useful approach to model the effects of surface roughness. Two time-domain numerical methods are considered: finite difference schemes (FDTD), and the transmission line modeling (TLM) method. An effective impedance model for random ground roughness defined by a roughness spectrum, called the SPM model, is exposed. The efficiency of this model for taking into account the mean effects of random roughness on sound pressure levels and for modeling the roughness-induced surface wave is shown, by comparing with results of TLM simulations of propagation above random rough grounds. The direct implementation of the SPM model as a boundary condition in both TLM and FDTD methods is then studied. This approach allows the modeling of ground roughness effects in numerical methods without having to mesh finely the ground roughness profile, allowing easier and faster computations, and more accurate predictions for future impact studies in environmental acoustics.

## 1. Introduction

The acoustical impact of industrial or transport installations on the environment is often estimated using simplified engineering methods. These methods need to be validated and fine tuned using reference results which can be experimentally or numerically obtained.

With the increase in computing power, the interest for the application of time-domain numerical methods to outdoor sound propagation has risen over the past years, *e.g.* see Refs. [1–4]. These methods are relevant as they can take into account most of the physical phenomena encountered during propagation, such as micro-meteorological effects (wind and temperature gradients, atmospherical turbulence) and ground effects, leading to very accurate numerical results for outdoor sound propagation.

Considering the ground effects, the interaction of the sound wave with a flat absorbing ground can be taken into account by transposing a frequency-domain boundary condition into the time-domain [5,6]. Realistic situations often involve irregular grounds with non-flat profiles. Ground irregularities with a topographic scale such as hills may be modeled using curvilinear coordinates solvers [7,8]. However natural grounds may also exhibit smaller geometry irregularities, whose characteristic size is inferior to the wavelength, known as ground roughness. Ground roughness produces a scattering of the sound wave that modifies the ground effect resulting from the interference between the direct and the ground-reflected wave. Ground roughness also leads to the formation of a surface wave [9,10].

This work focuses on the modeling of the effects of ground roughness in time-domain numerical methods for sound propagation, particularly the Finite Difference Time-Domain (FDTD) and Transmission Line Modeling (TLM) methods. Some difficulties arise when considering rough grounds, and the modeling of the ground profiles is not straightforward. First, the roughness may only be known statistically. Secondly, refining meshes at the boundaries in time-domain methods induces higher computation times (refining meshes could also be tricky regarding the introduction of phase error and artificial reflections at the transition zones between coarse and refined parts). To circumvent these difficulties, the

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effective impedance approach is considered: the effects of roughness on sound propagation are taken into account by considering a flat surface with a modified impedance boundary condition. Recently, the effective impedance approach has been used to model rough sea profiles in order to study numerically the propagation of noise generated by offshore wind farms [11]. Past works concern the effective impedance model proposed by Attenborough et al., that takes into account a roughness formed by small scatterers along the propagation path [12]. This model is known as the boss model, and the corresponding effective impedance is function of the geometry and spacing of the scatterers. Heuristic extensions of this model have been validated by reduced scale laboratory experiments [13,14,10] and outdoor measurements over agricultural surfaces such as plowed grounds [15]. Furthermore, the possibility and the interest to use this effective impedance model in time-domain numerical models was shown in Faure et al. [16].

In the present paper, a new effective impedance model for surface roughness is applied and tested for our specific application. Called the SPM model and originating from works in electromagnetism, it allows the modeling of the random roughness effects. The effective impedance is then expressed in function of the roughness spectrum of the height profile.

First, Section 2 exposes the boss model formalism and the SPM effective impedance model. The Gaussian roughness spectrum, which can be used to express the SPM effective impedance model, is also defined. In Section 3, TLM simulations of middle-range propagation above random rough grounds are performed, and the results are compared to analytical results with effective impedance in order to validate the accuracy of the SPM model for sound pressure levels predictions. The ability to model the roughness-induced surface wave using this effective impedance is also validated. Section 4 demonstrates the possibility and the interest to use the SPM effective impedance as a boundary condition in time-domain methods. The way to proceed for the implementation of the effective impedance in TLM and FDTD methods is exposed, and simulations are performed. Finally, in Section 5, the main results of this work are highlighted and concluding remarks are drawn.

## 2. Effective impedance models for random roughness

The effective impedance approach allows to model a rough ground by a perfectly flat ground with a modified impedance condition taking into account the effects of roughness on sound propagation. The effective impedance is expressed as a function of the roughness parameters. Then, using an effective impedance, sound levels above a rough ground can be estimated with simple models for propagation above a flat impedance ground with an homogeneous atmosphere, such as the well known Weyl-Van der Pol formula [17,18].

### 2.1. Boss model formulation for deterministic roughness

The boss effective impedance model proposed by Attenborough and Taherzadeh [19], based on Tolstoy's boss model [20] and Twersky's work [21], allows to calculate an effective impedance $Z_{\text{eff}}$ (or an effective admittance $\beta_{\text{eff}} = 1/Z_{\text{eff}}$) for a set of semi-cylindrical scatterers, as shown in Fig. 1.

The effects of roughness are taken into account as a correction to the surface admittance $\beta_S$, and $Z_{\text{eff}}$ is given by:

$$1/Z_{\text{eff}} = \beta_{\text{eff}} = \beta_S + \beta_R \tag{1}$$

where $\beta_S = 1/Z_S$. The base impedance $Z_S$ for the flat surface can be evaluated using several models from the literature, such as the Delany-Bazley or Miki models, the Zwikker and Kosten model or the Attenborough model (these common ground impedance models are described in Attenborough et al. [22] for example). Hard-backed layer correction (thickness effect) can also be applied for the expression of $Z_S$ [23]. The correction $\beta_R$ is function of the angles $\theta_i$ and $\phi$, the frequency, and other parameters depending of the scatterers' size, shape and spacing. These parameters and the exact formulation of $\beta_R$ can be found in Boulanger et al. [12]. The model is valid for wavelengths larger than the roughness characteristic size, such as $k_0 h < k_0 b \leqslant 1$ where $h$ is the scatterers' height ($h = a$ for the case of semi-cylindrical scatterers), and $k_0$ the wave number.

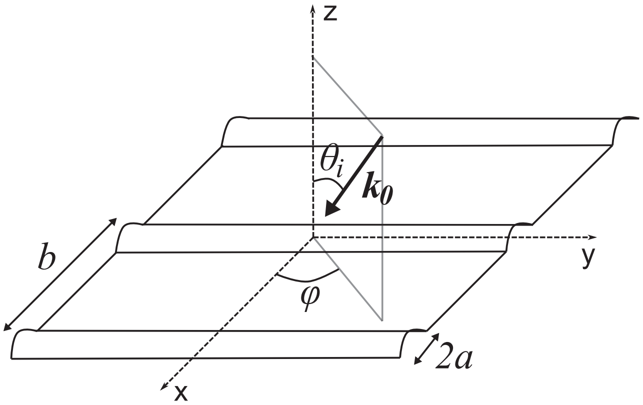

**Fig. 1.** Wave vector $\mathbf{k_0}$ incident to a surface containing cylinders of radius $a$ and mean center-to-center spacing $b$ [12].

### 2.2. SPM model for random roughness

An effective impedance model for sound propagation above hard randomly-rough surfaces was first developed by Watson and Keller [24,25]. This model is obtained using the Small Perturbation Method (SPM). It also found applications in the field of electromagnetic waves propagation above rough surfaces, such as the surface of the sea in the works of Brelet and Bourlier [26].

A 2D rough surface showing a small and slowly-varying roughness with $| k_0\zeta \cos(\theta_i) | < 1$ and $| \partial\zeta/\partial x | < 1$ is considered, as shown in Fig. 2. In this figure $\mathbf{k_0}$ and its modulus $| k_0 | = 2\pi f/c_0$ are respectively the wave vector and the wave number in the air, with $c_0$ the sound speed in the air, $\zeta(x)$ is the height profile, $\theta_i$ is the angle of incidence, $Z_0$ is the characteristic impedance of the air and $Z_S$ is the impedance of the surface.

Under this assumption, it is thus possible to perform finite expansions of the Neumann boundary condition and the Green's function for a point source above the profile $\zeta$. Then, the scattered field above the rough profile is modeled from a boundary integral formulation. A mean value of this integral is calculated using the Dyson equation and the Feynman diagram formalism [27]. Finally after using the Bourret approximation and some cumbersome manipulations [26,28], it is possible to derive an effective impedance from an equivalent reflection coefficient, in which the roughness effect is accounted for. This effective impedance is function of the roughness spectrum $W$ of the surface. This roughness spectrum is defined as the Fourier Transform of the autocorrelation function of the surface height profile $\zeta$ (also defined as the spectral density of $\zeta$), as follows:

$$W(k) = \int_{-\infty}^{+\infty} \exp(-2i\pi kx) C_\zeta(x) dx \tag{2}$$

with $C_\zeta(x) = \langle \zeta(x_1)\zeta(x_1 + x) \rangle$ the autocorrelation function of $\zeta(x)$.

The obtained effective impedance (or effective admittance) accounts for the mean effects of the random roughness on the