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Experimental investigation of a new two-microphone method for the determination of broadband noise radiation from ducts

Jonathan Melling, Jian Chen *, Phillip Joseph

Institute of Sound and Vibration Research, University of Southampton, Southampton SO17 1BJ, UK

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ABSTRACT

This paper experimentally investigates a new technique for measuring the modal amplitude distribution, sound power transmission and radiation, and far field directivity of the broadband noise from hard walled ducts. The innovative aspect of this method is that it only requires the measurements of the two-point complex coherence function between the acoustic pressures at two closely spaced points on the duct wall. This method is therefore very useful when direct measurements of sound power and directivity are not possible. This paper describes detailed measurements of the sound power spectrum and coherence function from a hard walled circular duct excited at one end by a diffuse sound field. The other open end is terminated within an anechoic chamber with which to measure the radiated sound field at 11 microphones distributed over a polar arc. Measurements of the complex coherence were made at the duct and used to infer the sound power spectrum and far field directivity. This paper demonstrates generally good agreement between direct measurements of sound power and directivity and those inferred from the coherence function. The method is restricted to broadband noise in large ducts in the frequency range where many modes are able to propagate and the modal amplitudes are mutually uncorrelated.

1. Introduction

Ducts, also known as waveguides, are able to efficiently transmit noise over large distances, which may then radiate from the end of the duct. Common examples are ventilation/exhaust ducts, automotive silencers, and turbofan aero-engines. Often one wishes to determine the far field directivity and sound power radiating from the duct open end, either as an index of transmission loss to assess the performance of silencer, or as a means of scaling the total noise level for predicting community noise annoyance.

In the case of an exhaust duct and a turbofan engine, for example, locating microphones in the far field is difficult. In the case of turbofan engines very large anechoic facilities are needed. Without these facilities, therefore, an alternative technique the far-field directivity and sound power may be inferred from in-duct acoustic pressure measurements. In-duct measurement techniques have been developed using microphone arrays to localise the broadband sources in the duct [1–3] using acoustic imaging methods, such as beamforming. Measurement techniques have also been developed to estimate the far-field directivity from acoustic pressure measurements made on the duct wall based on a decomposition of the sound field into its propagating modal components. The amplitudes of each mode are first determined by a microphone phased array. Next the pattern of individual mode propagating to the far-field is predicted by the amplitude of the corresponding in-duct mode.

Measurement techniques for the modal decomposition of tonal noise in the duct has been developed in the early 1970s [4] by inverting a matrix of modal response functions based on a modal propagation model. The modal decomposition of broadband noise is more problematic however since the mode amplitudes are partially coherent and all modes are potentially excited. In a typical aero-engine duct, for example, the number of propagating modes can easily exceed 100 at the blade passing frequency. One current method to determine the amplitude of each mode requires a large microphone array in the duct comprising many rings of microphones. In general, at least as many microphones as modes are required to deduce all mode amplitudes. Researchers at Boeing [5] developed a circumferential row array with a large number of microphones to evaluate sound pressure as a function of azimuthal wavenumber or spinning mode order. Later, researchers at Boeing [6] designed more sophisticated ring arrays in the inlet, inter-stage and bypass sections of an aero-engine duct to decompose the acoustic pressure at the wall sound pressure level into spinning modes. However, neither work attempted to relate the in-duct

* Corresponding author.
E-mail addresses: chenjian_scu@aliyun.com, jc2d14@soton.ac.uk (J. Chen).

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Nomenclature

a	duct radius	S_{12}	Pressure Cross Spectral Density at the duct wall
$\overline{a_{\pm}^2}$	normalised mean square mode amplitude distribution for positive and negative propagating modes	S_{ff}	Pressure Power Spectral Density of far-field radiation
c	speed of sound	$S(\omega)$	frequency-dependent source strength
f_{sch}	schroeder frequency	$\lvert T(\omega)\rvert^2$	energy transmission coefficient
k	acoustic wavenumber	T_{60}	reverberation time
p	acoustic pressure	V	source room volume
x_D	distance along duct of measurement	$W(\omega)$	transmitted power
Δx	in-duct microphone separation distance	α	cuton ratio
A	duct area	θ	polar co-ordinate of far-field observer
A_{mn}	pressure amplitude of mode	ρ	density of air
L	length of duct	ω	angular frequency
N	total number of propagating modes at a particular frequency	$\hat{\omega}$	non-dimensional frequency
$\lvert R_r(\omega)\rvert^2$	energy reflection coefficient	ψ_{12}	complex coherence function
R	far-field radial position from centre of the duct	Ω^{+}	sound power factors
S_{11}	Pressure Power Spectral Density at the duct wall	$\Gamma\,(\hat{\omega})$	Hanning window function

modal decomposition to the far-field sound radiation. Enghardt et al. [7,8] has recently proposed an in-duct measurement technique to decompose the broadband sound field into its constituent modes at a maximum frequency corresponding to about 150 propagating modes. No attempt was made to predict the far field radiation, however. Another method [9] is based on an axial array of microphones along the duct wall. Conventional beamforming was applied to the array to estimate the mode amplitude distribution versus the modal propagation angle, which was then used to infer both the total sound power and far field directivity. However, this method requires many microphones arranged along a long length of duct (many acoustic wavelengths).

As mentioned above, using large arrays of microphones have the disadvantage that they occupy a large space, which is not always available as well as being costly since it requires good quality microphones with stable phase characteristics as well as a data acquisition system comprising a large number of channels. To overcome these difficulties a new method has recently been developed by Joseph et al. [10,11] in which the mode amplitude distribution, transmitted and radiated sound power, and far field directivity can be estimated from measurements of only the complex coherence function between the acoustic pressure at two closely spaced positions at the duct wall. The method makes a number of crucial assumptions about the sound field, which have so far *not* been validated experimentally. This paper presents an experimental investigation into the accuracy and validity of this two microphone method and validate experimentally some of the main simplifying assumptions behind the technique.

2. Measurement theory

Before discussing the experimental method and measurement procedure we first outline the theory underlying the measurement principle. A fuller derivation of the underlying theory is presented in Appendix A and the detailed theoretical development presented in Ref. [10, https://www.acoustics.asn.au/conference_proceedings/INTERNOISE2014/papers/p949.pdf].

2.1. Mode amplitude distribution

The theory in Ref. [10] includes the effects of a uniform mean flow. Here the effects of flow are ignored for consistency with the no-flow measurements presented in Sections 3 and 4.

Above its cutoff frequency, at a single (angular) frequency ω, a single mode propagating along the duct of pressure amplitude A_{mn} is described by

$$p_{mn}^{\pm}(\boldsymbol{y},x) = e^{-i\omega t}A_{mn}^{\pm}\Psi_{mn}(\boldsymbol{y})\,e^{\pm i\alpha_{mn}kx} \tag{1}$$

In this equation the superscript "+" and "−" refers to the modes propagating in the positive (away from the source) and negative x directions (towards the source), respectively, m and n denote the usual mode indices of the propagating azimuthal mode and radial mode respectively, $\boldsymbol{y} = (r, \theta)$ is a position vector on the duct cross sectional area with radial and azimuthal coordinates r and θ, $\Psi_{mn}(\boldsymbol{y})$ denotes the modal shape function with the normalization property, $A^{-1}\int_A |\Psi_{mn}(\boldsymbol{y})|^2 dA(\boldsymbol{y}) = 1$ where A is the duct cross sectional area, and $k = \omega/c$ is the free space wavenumber and c is the sound speed. Of central importance in Eq. (1) is the parameter α_{mn}, which we shall call the modal cut-on ratio, given by,

$$\alpha_{mn} = \sqrt{1-(\kappa_{mn}/k)^2} \tag{2}$$

where κ_{mn} is a set of eigenvalues that are characteristic of the duct cross section such that the corresponding mode shape functions Ψ_{mn}, defined by $(\nabla_{\perp}^2 + \kappa_{mn}^2)\Psi_{mn}(\boldsymbol{y}) = 0$, also satisfy the duct-wall boundary conditions. In a hard wall duct, κ_{mn} takes the values $\kappa_{mn} = j'_{mn}/a$ where j'_{mn} denotes the n^{th} stationary value of the Bessel function of order m. The cut-on ratio α_{mn} is central in what follows and takes values between $\alpha_{mn} = 0$ precisely at the modal cutoff frequency $\omega = \omega_{mn} = \kappa_{mn}c$, and tends to $\alpha_{mn} = 1$ as $\omega/\omega_{mn} \to \infty$, corresponding to modes well above cuton.

The significance of the cuton ratio to duct acoustics, or its related quantity cuton ratio $(1-\alpha_{mn}^2)^{-1} = k/\kappa$, was highlighted in the work of Rice [12] and Joseph et al. [13]. It is an important quantity in duct acoustics since different modes with the same cuton ratio have similar transmission and radiation characteristics, Rice [12]. Cuton ratio is uniquely related to the angle θ_{mn} through $\alpha_{mn} = cos\theta_{mn}$ [14], with which the mode propagates along the duct relative to the axis. It has also been shown that θ_{mn} equals to the angle radiated most strongly to the far field (the angle of the main lobe), Rice [12]. Lastly, it has been shown that the mode amplitude distribution for many physical source distributions is a smoothly varying function of α_{mn}.