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Transfer matrix methods for sound attenuation in resonators with perforated intruding inlets

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ABSTRACT

A resonator with perforated intruding inlet (PII) is a superior silencer element, since the use of perforated inlet extensions can dramatically improve the acoustic performance. In this work, both a one-dimensional (1D) and a two-dimensional (2D) transfer matrix methods are developed to predict the transmission loss of the resonator without considering the mean flow. Based on the two groups of comparisons with tests, it is found out that the applicability of 1D method is limited by the resonator geometry even when the frequency is below the cut-off value of plane wave. Whereas the 2D approach is much more accurate while predicting the transmission losses within entire frequency range. Subsequently, five groups of resonators are chosen to determine the effects of structure parameters to transmission loss based on the 2D approach. The resonant frequency decreases and more resonant peaks appear when the length of inlet extension increases. A higher perforation rate leads to a shift of resonant peak towards higher frequencies. Besides, better acoustic performance could be obtained with the perforation being properly designed. Reducing the inlet/outlet radius can obviously improve the transmission loss without changing the frequency of resonant peak. The theories and conclusions in this study can be used for the design and optimization of resonators in various engineering applications.

1. Introduction

Recently, research on traditional silencers (expansion chamber muffler [1], quarter wavelength tube [2], concentric perforated resonator [3], etc.) has been continuously addressed. However, the research on resonators with perforated intruding inlets (PII) which could distinctly improve the acoustic performance has been neglected. As a promising kind of silencer element, a resonator with PII has the advantages of a compact structure and a desirable sound attenuation performance especially at mid and high frequencies [4]. Compared to the extended-tube resonator, additional perforations can effectively adjust and probably widen the frequency range of sound attenuation. Considering the design and engineering application of resonators with PII, it is significant to develop analytical approaches applied in the prediction for the transmission loss (TL).

The transfer matrix method [5] (TMM) based on plane wave theory is to obtain the four-pole parameters of a resonator, which are used to determine the TL. Chiu [6] used a one-dimensional (1D) TMM to calculate the TL of a muffler with perforated intruding inlet. In addition, with the effect of higher order modes excluded, the 1D approach is limited to the cut-off frequencies of plane wave [7]. Finite element method (FEM) takes more geometry details and three-dimensional effects into account [8], hence it can predict the acoustic performance more precisely. However, the model design and calculation process are time-consuming. It is also inconvenient to optimize the structure parameters if the acoustic performance is not satisfying. Therefore, apart from the 1D TMM, it is also necessary to develop a theoretical method, which is simultaneously accurate and efficient to calculate the TL of resonators with PII.

A two-dimensional (2D) method is applicable to calculate the TL for axisymmetric resonators. Selamet [9,10] predicted the TLs of both a single-chamber and a dual-chamber circular expansion muffler with extended inlet/outlet using a 2D weighted-integration method. However, the acoustic continuity equations to be solved in the approach will be complicated when many chambers are connected, since the transfer matrixes of the silencer elements are not considered and all continuity equations must be solved at one time. In recent years, the 2D approach was mostly applied to dissipative mufflers with single chambers [10–12]. Until

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Nomenclature

S_n	modal amplitudes in region S (A, B, C, D, E)	L	total length of resonator chamber
c	sound speed in air	P	acoustic pressure
hd	diameter of the perforations	r_1	radius of the inlet tube
ht	thickness of the inner tube	r_2	radius of the outlet tube
d_1	diameter of the inlet tube	R	radius of the chambers
d_2	diameter of the outlet tube	T	transfer matrix
D_o	diameter of the chambers	U	particle velocity
f	frequency	v	air viscosity
J_0, J_1	Bessel functions of the first kind of order 0 and 1	x, x_1, x_2, x_3	axial coordinates
k_0	sound wave number in air	Y_0, Y_1	Bessel functions of the second kind of order 0 and 1
$k_{x,S,n}$	axial wave number in region S (A, B, C, D, E)	α	end correction coefficient
$k_{r,S,n}$	radial wave number in region S (A, B, C, D, E)	ζ	perforation impedance
li	length of inlet extension	σ	porosity
lm	length of perforation area	ρ	air density
lo	length of expansion chamber	$\varnothing_S^n(r)$	eigenfunctions in region S (A, B, C, D, E)

now, an effective 2D analytical method for TL calculation of resonators with PII has not been developed yet. Besides, the transfer matrix of the resonator, which is commonly used in the TL prediction of multi-chamber silencers [14], has not been derived through 2D approach.

The objective of the present work is to investigate the acoustic modeling of a resonator with PII, which has the advantages of a compact structure and a superior acoustic performance. Firstly, referring to an existing study, a 1D TMM without considering the mean flow is derived. In order to predict the acoustic performance more precisely, a 2D analytical method using direct integration is developed to calculate the pressure magnitudes for incident and reflected waves in the resonator, which are subsequently used to determine the four pole parameters of the transfer matrix. The applicability of the two TMMs is discussed in details through the comparisons to FEM and tests. To further study the characteristics of the resonator with PII, five groups of resonators are selected to evaluate the effects of structure parameters to the TL based on the 2D approach, including the length of perforated/non-perforated inlet extension, perforation rate and inlet/outlet radius.

2. Analytical methods

2.1. One-dimensional transfer matrix method

A resonator with PII consists of four kinds of acoustic propagation sections as shown in Fig. 1, including

(1) Straight tube, namely from point 1 to point 2 and from point 6 to point 7.
(2) Concentric perforated tube, namely from point 2, 3 to point 4, 5.
(3) Sudden expansion at the open-ended inlet, namely from point 4, 5 to point 6.
(4) Sudden retraction at the outlet, namely from point 7 to point 8.

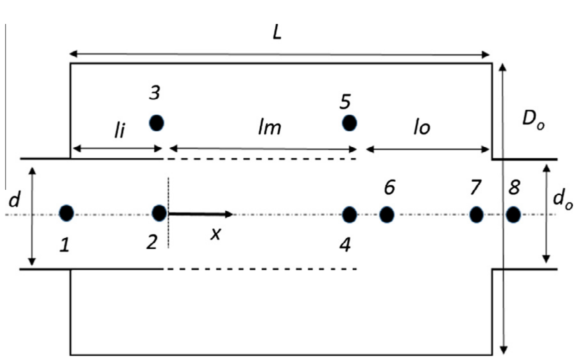

Fig. 1. The configuration and acoustic points of a resonator with PII.

The total chamber length L is divided into an extended inlet of length li, a perforated tube of length lm, and an expansion chamber of length lo. Diameters of the inlet, outlet, and outer chamber are d, d_o, and D_o. The 1D TMM is to calculate the total transfer matrix of the resonator through multiplying transfer matrixes of every connected acoustic sections. The transfer matrixes for the straight tubes can be expressed as

$$\begin{bmatrix} p(1) \\ \rho cu(1) \end{bmatrix} = \begin{bmatrix} \cos(k_0 li) & j\sin(k_0 li) \\ j\sin(k_0 li) & \cos(k_0 li) \end{bmatrix} \begin{bmatrix} p(2) \\ \rho cu(2) \end{bmatrix} = [T1] \begin{bmatrix} p(2) \\ \rho cu(2) \end{bmatrix} \tag{1}$$

$$\begin{bmatrix} p(6) \\ \rho cu(6) \end{bmatrix} = \begin{bmatrix} \cos(k_0 lo) & j\sin(k_0 lo) \\ j\sin(k_0 lo) & \cos(k_0 lo) \end{bmatrix} \begin{bmatrix} p(7) \\ \rho cu(7) \end{bmatrix} = [T4] \begin{bmatrix} p(7) \\ \rho cu(7) \end{bmatrix} \tag{2}$$

where p is the sound pressure; ρ is the air density; c is the sound speed in air; u is the particle velocity; $k_0 = 2\pi/f$ is the sound wave number in air; f is the sound frequency. For the perforated region, the wave propagation functions in the inner tube and outer chamber are [15]

$$\begin{cases} \frac{\partial^2 p_i}{\partial x^2} - \left(\frac{4jk_0}{d\xi} - k_0^2\right) p_i + \frac{4jk_0}{d\xi} p_o = 0 \\ \frac{\partial^2 p_o}{\partial x^2} - \left(\frac{4jdk_0}{(D_o^2 - d^2)\xi} - k_0^2\right) p_o + \frac{4jdk_0}{(D_o^2 - d^2)\xi} p_i = 0 \end{cases} \tag{3}$$

where p_i, p_o, u_i and u_o are respectively the sound pressure and particle velocity of the inner tube and outer chamber; ξ (see Appendix A) is the acoustical impedance of the perforated tube. Eq. (3) can be written as a state function

$$\begin{bmatrix} p_i' \\ p_o' \\ p_i \\ p_o \end{bmatrix}' = [T] \begin{bmatrix} p_i' \\ p_o' \\ p_i \\ p_o \end{bmatrix} \tag{4}$$

The related solution can then be written as

$$\begin{bmatrix} p_i' \\ p_o' \\ p_i \\ p_o \end{bmatrix} = [\psi][C_1 e^{\lambda_1 x}, C_2 e^{\lambda_2 x}, C_3 e^{\lambda_3 x}, C_4 e^{\lambda_4 x}]^T \tag{5}$$