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Mutual effects of thermal radiations and thermal stratification on tangent hyperbolic fluid flow yields by both cylindrical and flat surfaces

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ABSTRACT

The characteristics of tangent hyperbolic fluid flow in the presence of both thermal radiation and thermal stratification effects are not discussed until now especially by way of an inclined cylindrical surface. Therefore, an endeavour as a comparative study is reported to offered numerical solution for tangent hyperbolic fluid model towards both cylindrical and flat stretching surfaces. The physical situation which also includes the noteworthy effects namely, stagnation point region, mixed convection, magnetic field individualities and heat generation is translated in terms of partial differential equations and for solution purpose a computational algorithm is executed. The obtained results are provided by way of both graphical and tabular outcomes. It is observed that the temperature profile is an increasing function of thermal radiation parameter but opposite trends are noticed for positive values of thermal stratification parameter. In addition, straight line approximation is accomplished to scrutinize the impact of mixed convection and an inclination effects on skin friction coefficient while the impact of thermal radiation and thermal stratification parameters are provided on heat transfer rate.

1. Introduction

The study of fluids having non-Newtonian characteristics subject to stretching surfaces in the presence of heat transfer properties always remains a topic of great interest for scientists and engineers. This is because of numerous applications of non-Newtonian fluids namely, hot rolling, wire drawing, polymer sheet, petroleum production and construction of paper production to mention just a few. By considering these involvements majority of investigators explored (see Refs. [1–14]) that the viscous fluids are not primary suitable in contrast to non-Newtonian fluid models. To be more specific, we have non-linear relationship between shear stress and shear rate which leads to flow diversity of non-Newtonians fluid models. In general, due to uncertainty in rheological features of non-Newtonian fluid models the single constitutive equation is not enough to conferred the relation of shear stresses and shear rate. In this context, various model are proposed namely, White-Metzner, Rolie-Poly, Blatter fluid model (1995), FENE-CR (1988), Giesekus fluid model (1982), Phan-Thien-Tanner fluid model (1978), Johnson-Tevaarwerk (1977), Johnson-Segalman fluid model (1977), Carreau-Yasuda fluid model (1972), Carreau fluid model (1972), FENE-P (1966), Cross fluid model (1965), Seely fluid model (1964), Kaye-Bernstein-Kearsley-Zapas (K-BKZ, 1963), Sisko fluid model (1958), Criminale-Ericksen-Filbey (1957), Rivlin-Ericksen (1955), Glen fluid model (1955), Oldroyd-A (1950), Oldroyd-8 constants (1950), Oldroyd-B fluid model (1950), Reiner-Rivlin fluid model (1945), Generalized Burgers (1939), Burgers fluid model (1939), Eyring fluid model (1936), Williamson fluid model (1929), Ostwald-de

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Waele power law model (1923), Bingham Herschel-Bulkly (1922), Barus fluid model (1893), Maxwell fluid model (1867), etc. In addition, out of these non-Newtonian fluid models, Cross model, power law model, Carreaus model, Ellis model and tangent hyperbolic fluid model are known by pseudo-plastic features. The pseudo-plastic fluids are very useful in various engineering applications namely, emulsions, coated photographic films, polymers and solution with relatively higher molecular weight to mention just a few. Tangent hyperbolic fluid model describes the flow of shear thinning non Newtonian fluids. As an example the industrial and biological liquids that obey the tangent hyperbolic fluid are polymers, melts/solution, ketchup blood, paint, whipped cream etc. Akbar et al. [15] discussed the numerical solutions of magneto hydrodynamic boundary layer flow of tangent hyperbolic fluid towards a stretching sheet. They used fourth and fifth order Runge-Kutta-Fehlberg technique to solve the system of ordinary differential equations. Naseer et al. [16] studied the boundary layer flow of tangent hyperbolic fluid over a vertical exponentially stretching cylinder by using Runge-Kutta-Fehlberg technique. Akram et al. [17] presented the consequence of nano-fluid on peristaltic transport of a tangent hyperbolic fluid model in the occurrence of magnetic field. They found that the velocity of the fluid enhances for large values of magnetic field. Nadeem et al. [18] found numerical solution and provided the effects of nanoparticles on the peristaltic motion of tangent hyperbolic fluid in an annulus using fourth and fifth order RK–Fehlberg method. Abbas et al. [19] examined the three dimensional peristaltic flow of tangent hyperbolic fluid in non-uniform channel having flexible walls. Hayat et al. [20] highlighted tangent hyperbolic nanofluid flow due to convectively heated Riga plate with variable thickness. Hayat et al. [21] also studied the tangent hyperbolic fluid flow of MHD effect with slip conditions and Joule heating in an inclined channel. Nadeem and Shahzadi [22] studied the effects of an induced magnetic field on tangent hyperbolic nanofluid in a curved channel.

The flow regime having thermal layers due to combination of fluids with distinct densities is termed as thermally stratified fluid flow. Such a formation is also known as temperature stratification or thermal stratification. As an application ratio of hydrogen and oxygen are balanced by means of stratification phenomena to reduce the growth rate of species in the case of lakes and ponds while oceanography, agriculture, geophysical flows, astrophysical involvements and various chemical processes claims the prime role of stratification either it is solutal stratification or thermal. Plenty of researcher [23–34] made different attempts to identify the individualities of stratification phenomena by entertaining various physical effects. Thermal radiations are emitted as an energy in the form of electromagnetic radiations. These radiations travel equivalent to the speed of light and for its propagation we do not required any type of medium. As an example, radiator, elements heating on stove, fire, and light through sun are the source of thermal radiations. An influence of thermal radiations subject to boundary layer flows is an essential part to study because of practical applications in the field of engineering like furnace designing, glass production, gas cooled nuclear reactors, power plants, polymer processing, and propulsion system. Therefore, while encountering boundary layer flows the effects of thermal radiations are need to be evaluated by considering Rosseland approximation in energy equation to identified radiative heat flux properties. Many researchers admits the importance of involvement of thermal radiations and proposed ultimate consequences [35–47].

From the above reported literature it is concluded that few attempts are available in literature with accuracy to encountered tangent hyperbolic fluid flow yields by stretching surfaces. To be more specific, the combined aspects of thermal radiation and thermal stratification effects are not reported on tangent hyperbolic fluid flow towards an inclined stretching surfaces. Therefore, we have assumed a tangent hyperbolic fluid flow yields by both an inclined cylindrical and flat stretching surfaces. The fluid flow is achieved by considering no slip condition. Flow exploration is conceded with noteworthy effects namely, stagnation point region, mixed convection, magnetic field individualities and heat generation. The physical illustration is translated in terms of PDE's. A computational algorithm is executed to report numerical solution. The effect logs of an involved physical parameters are identified on dimensionless velocity and temperature distributions through graphical and tabular outcomes. It is noticed that the effects of an involved parameters namely, mixed convection parameter, Weissenberg number, thermal radiation parameter and thermal stratification parameter are enriched for cylindrical surface as compared to flat surface. The impact of the mixed convection, an inclination effects on skin friction coefficient while the impact of thermal radiation thermal stratification parameters on heat transfer rate is presented by way of straight line curve fitting scheme.

2. Mathematical description

A steady laminar magneto-hydrodynamic an incompressible boundary layer stagnation point flow of tangent hyperbolic fluid with zero pressure gradient is considered. The fluctuation velocity gradients due to viscous stresses are assumed to small in a laminar boundary layer flow tangent hyperbolic fluid, so that the viscous dissipation effect is neglected. The fluid flow is entertained through no slip condition that is the stretching velocity of an inclined cylindrical surface corresponds the velocity of fluid particles. Further, flow field is manifested with mixed convection, thermal stratification, heat generation and thermal radiation phenomena. The strength of temperature near the cylindrical surface is supposed to be higher than the ambient fluid. As far as the geometric configuration is concerned the axial line of cylinder is aligned towards $\widetilde{X}$-axis and radial direction is aligned perpendicular to the fluid flow and considered as $\widetilde{R}$-axis. In the field of fluid mechanics the generally accepted differential equations are momentum and energy. They are sufficient to illustrates the flow fields characteristics. So, that the finalized forms of these equations subject to boundary layer approximation are given as follow:

$$\frac{\partial(\widetilde{R}\widetilde{U})}{\partial\widetilde{X}} + \frac{\partial(\widetilde{R}\widetilde{V})}{\partial\widetilde{R}} = 0, \tag{1}$$