Commun Nonlinear Sci Numer Simulat 53 (2017) 142–153

Contents lists available at ScienceDirect

Commun Nonlinear Sci Numer Simulat

journal homepage: www.elsevier.com/locate/cnsns

Research paper

On the mathematical modeling of soccer dynamics

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ARTICLE INFO

Article history:
Received 9 March 2017
Revised 20 April 2017
Accepted 24 April 2017
Available online 25 April 2017

Keywords:
Discrete systems
Complex systems
Sport dynamics
Power law
Multidimensional scaling
Fractional calculus

ABSTRACT

This paper addresses the modeling and dynamical analysis of soccer teams. Two modeling perspectives based on the concepts of fractional calculus are adopted. In the first, the power law behavior and fractional-order integration are explored. In the second, a league season is interpreted in the light of a system where the teams are represented by objects (particles) that evolve in time and interact (collide) at successive rounds with dynamics driven by the outcomes of the matches. The two proposed models embed implicitly details of players and coaches, or strategical and tactical maneuvers during the matches. Therefore, the scale of observation focuses on the teams behavior in the scope of the observed variables. Data characterizing two European soccer leagues in the season 2015–2016 are adopted and processed. The model leads to the emergence of patterns that are analyzed and interpreted.

1. Introduction

Sports teams are complex systems (CS) constituted by players driven by a simple common objective: winning collectively the competition challenges. During the match, the individual skills of the players, their interactions, and the inputs from the environment give rise to the emergence of an intricate collective behavior. The resulting time-space patterns have been analyzed by the mathematical and computational tools adopted for tackling dynamical systems [1–10].

Besides the behavior of a team in a given match, the long-term competitiveness of distinct teams within the same league has also been investigated, namely by data envelopment analysis [11], complex networks [12], graphs and centrality measures [13], and multidimensional scaling [14], among others [15]. The main idea is to overcome the limitations of traditional analysis, that measures competitiveness focusing on simple statistics, such as win ratios, point accumulation, or historical dominance of some teams [16,17].

Soccer (also known as association football, or football) is one of the most popular team sports around the world. It involves more than 250 million players in about 200 countries [1,18]. The game is played by 2 teams, composed of 11 players each, on a rectangular field with a goal placed at each end. The objective of the game is to score by getting a spherical ball into the opposing goal. The 10 field players can maneuver the ball using any part of the body except hands and arms, while the goalkeeper is allowed to touch the ball with the whole body, as long as he/she stays in his/her penalty area. Otherwise, the rules of the field players apply. The match has 2 periods of 45 minutes. The winning team is the one that scores more goals by the end of the match. Most European leagues are played by groups of teams that compete during

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http://dx.doi.org/10.1016/j.cnsns.2017.04.024
1007-5704/

Table 1
"Premier League" and "La Liga" final standings in season 2015–2016.

"Premiership"			"La Liga"		
Rank	Team	Acronym	Rank	Team	Acronym
1	Leicester City	Le	1	FC Barcelona	Ba
2	Arsenal FC	Ar	2	Real Madrid	RM
3	Tottenham Hotspur	To	3	Atlético Madrid	AM
4	Manchester City	MC	4	Villarreal CF	Vi
5	Manchester United	MU	5	Athletic Bilbao	AB
6	Southampton FC	So	6	Celta Vigo	CV
7	West Ham United	WH	7	Sevilla FC	Se
8	Liverpool FC	Li	8	Málaga CF	Ma
9	Stoke City	SC	9	Real Sociedad	RS
10	Chelsea FC	Ch	10	Real Betis	RB
11	Everton FC	Ev	11	UD Las Palmas	LP
12	Swansea City	SC	12	Valencia CF	Va
13	Watford FC	Wa	13	Espanyol Barcelona	EB
14	West Bromwich Albion	WB	14	SD Eibar	Ei
15	Crystal Palace	CP	15	Deportivo La Coruña	DC
16	AFC Bournemouth	Bo	16	Granada CF	Gr
17	Sunderland AFC	Su	17	Sporting Gijón	SG
18	Newcastle United	NU	18	Rayo Vallecano	RV
19	Norwich City	NC	19	Getafe CF	Ge
20	Aston Villa	AV	20	Levante UD	Le

a given number of rounds. All teams start with zero points and, at every round, a {victory, draw, defeat} worths a "quanta" of {3, 1, 0} points. By the end of the last round, the team that accumulated more points is crowned champion.

This paper studies soccer teams competing within the same league season. Two alternative modeling approaches based on the fractional calculus concepts are proposed. In the first, the use of power law (PL) trendline approximations is explored, reflecting the fractional-order behavior of the teams results. The second model is inspired on the dynamics of a system of particles with punctual masses that interact in a 1-dimensional discrete-space (of results) with impacts at discrete-time instants (rounds). In both perspectives, a league season is interpreted as a system where the elemental objects are the teams. These objects interact at each round and evolve driven by the results obtained at the matches. This viewpoint overlooks the phenomena that occur within each individual match, but emphasizes the behavior of the teams along the season. The emerging patterns in time and space are discussed and interpreted.

Bearing these ideas in mind, this paper is organized as follows. Sections 2 describes the experimental dataset. Section 3 develops the two alternative modeling perspectives and analyses the teams behavior in 2 top European soccer leagues in season 2015–2016. Finally, Section 4 outlines the main conclusions.

2. Description of the dataset

Data for worldwide soccer is available at http://www.worldfootball.net/. The database contains information about national leagues and international competitions. For the national leagues, the results of the matches are organized in a per season basis. For each match we have the names of the home and away teams, the goals scored, the points won, and the date of the match, among other information.

We consider the 2015–2016 season of the 2 top national European leagues, namely the English "Premier League" and the Spanish "La Liga". The "Premier League", or "Premiership", was established in 1992 as the most important league of the English association football. It is contested by 20 teams and adopts a system of promotion and relegation with the "Football League". The "Premier League" is now the most popular football league in the world, and the one that registers the highest stadium occupancy among all soccer leagues in Europe. "La Liga" started in 1929 as the top division of the Spanish soccer league system. It has been considered by UEFA the strongest league in Europe for the last five years. Since 1997 "La Liga" is contested by 20 teams. At the end of every season, the three lowest placed teams are relegated to the "Second Division", and are replaced by the top two teams of this league plus the winner of a play-off competition.

In season 2015–2016 the "Premier League" and "La Liga" champions were Leicester City and FC Barcelona, respectively. The final standings are summarized in Table 1.

3. Data analysis and results

In this section we consider a league season as a system where the teams are elemental constitutive objects. In Section 3.1, we approximate the system time-space discrete dynamics by means of PL functions. In 3.2, we formulate and analyze an abstract mathematical model inspired on the motion of a collection of particles with impacts to describe the system behavior.