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Short communication

Persistence of nonlinear hysteresis in fractional models of Josephson transmission lines

J.E. Macías-Díaz*

Departamento de Matemáticas y Física, Universidad Autónoma de Aguascalientes, Avenida Universidad 940, Ciudad Universitaria, Aguascalientes 20131, Mexico

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ABSTRACT

In this note, we depart from a model describing the transmission of electric currents in Josephson-junction chains, and provide a fractional generalization using Riesz discrete differential operators. The fractional model considered has generalized Hamiltonian- and energy-like functionals. The model and the energy functionals are fully discretized in order to investigate numerically the complex dynamics of the system when a sinusoidal perturbation at one end of the chain is imposed. As one of the most important results in this report, we establish the persistence of the nonlinear phenomena of supratransmission and infratransmission in Riesz fractional chains. Nonlinear hysteresis loops are obtained numerically for some values of the order of the fractional derivative, and numerical simulations of the propagation of monochromatic wave signals through the transmission line are presented using the nonlinear bistability of the system.

1. Introduction

Nonlinear supratransmission is a phenomenon that was firstly investigated in chains of harmonic oscillators [1]. This process consists in the sudden increase in the amplitude of wave signals that propagate in a nonlinear chain driven at one end by a harmonic disturbance, and it has been found in Klein–Gordon [2] and Fermi–Pasta–Ulam chains [3] amongst many other nonlinear media [4–7]. Historically, the study of energy transmission in nonlinear wave equations has been an interesting topic of investigation [8]. These models have applications in the description of data transmission in optical fibers [9] and in the study of the self-induced transparency of systems subject to a high-energy incident laser pulse [10]. More generally, the behavior of continuous media subject to a wave radiation is a fundamental problem that has potential applications in many nonlinear systems [11]. In the case of discrete nonlinear chains, supratransmission was observed in systems of pendula coupled through springs [12]. Later on, applications to the design of digital amplifiers of ultra weak signals [13] and light detectors sensitive to very weak excitations [14] have been proposed. Further applications to optical waveguide arrays using the discrete nonlinear Schrödinger equation [3], the realization of light filters [15] and the propagation of binary signals in undamped or weakly damped mechanical chains of oscillators [16] have been realized. Some recent papers have been devoted to understand deeply this phenomenon under various physical scenarios, and to propose further applications [17].

* Corresponding author.
E-mail address: jemacias@correo.uaa.mx

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In addition to those articles, there are many reports which investigate physically and mathematically the occurrence of the phenomenon of nonlinear supratransmission [18]. However, still many questions remain unanswered while other avenues of research open up with the development of new empirical or mathematical tools. For instance, models that consider fractional derivatives have attracted the attention of many researchers in recent years [19,20]. Classical systems that include derivatives of integral order have been extended in this way using various inequivalent approaches [21], and interesting results have been derived in the way. Obviously, the inclusion of nonlocal differential operators provides more accurate descriptions in some physical contexts [22], and the question whether nonlinear supratransmission is present in fractional descriptions of Josephson transmission lines naturally arises in this form. The purpose of this letter is to investigate the presence of nonlinear bistability in a fractional generalization of the classical Josephson transmission line. Motivated by some preliminary studies on Riesz space-fractional sine–Gordon equations [23], we will provide a Riesz fractional extension of the model investigated in [24] for derivative orders in the interval (1, 2), and show that the process of nonlinear hysteresis persists in these new systems. In particular, the coalescence of supratransmission and infratransmission in fractional chains will be exhibited through simulations. As expected, this bistable regime will be applied to the transmission of wave signals from one end of the system. Our findings will extend various results on the controlled propagation of information through nonlinear regimes.

This communication is divided as follows. In Section 2, we recall the mathematical model describing the transmission of electric signals in a discrete chain consisting of Josephson junctions coupled through superconducting wires. The Hamiltonian of the undamped system as well as the energy operator are also recorded therein. Section 2 provides also a space-fractional generalization of the classical Josephson system using discrete Riesz differential operators. Suitable Hamiltonian- and energy-like operators are cited in that stage [25]. Section 3 is devoted to exhibit numerically the presence of nonlinear supratransmission and nonlinear bistability in the fractional system of interest. We also simulate in that section the propagation of binary signals in the fractional Josephson transmission line using the nonlinear bistability of the system. A discussion motivated by an interesting question from one of the reviewers is given therein. This letter closes with a section of concluding remarks and directions for future investigation.

2. Preliminaries

2.1. Josephson transmission line

Throughout this manuscript, we let $N \in \mathbb{N}$, and define the sets $J_N = \{1, 2, \ldots, N\}$ and $\bar{J}_N = J_N \cup \{0, N+1\}$. The present work is motivated by the investigation of the nonlinear phenomena of supratransmission and bistability in finite chains consisting of Josephson junctions coupled through superconducting wires. Concretely, assume that γ and c are nonnegative real numbers, and let $u_n \colon [0, \infty) \to$ be a twice differentiable function for each $n \in J_N$. The model describing the transmission of electric signals in a chain of Josephson junctions is given by

$$\begin{aligned} &\ddot{u}_n(t) - c^2 \Delta u_n(t) + \gamma_n \dot{u}_n(t) + \sin u_n(t) = \mu, \quad \forall n \in J_N, \\ &\text{such that} \quad \begin{cases} \dot{u}_n(0) = u_n(0) = 0, & \forall n \in J_N, \\ u_0(t) - u_1(t) = \dfrac{\phi(t)}{c^2}, & \forall t > 0, \\ u_{N+1}(t) - u_N(t) = 0, & \forall t > 0, \end{cases} \end{aligned} \tag{2.1}$$

where $\gamma_n = \gamma$ for every $n < N$, and $\gamma_N = \gamma + 1/R$. In this context, the number $R > 0$ is called the *output reading resistance* of the system, and is related to the output current intensity I through Ohm's law, namely, $I = \dot{u}_N / R$. In turn, the operator Δ is defined for each $n \in J_N$ by

$$\Delta u_n(t) = u_{n+1}(t) - 2u_n(t) + u_{n-1}(t). \tag{2.2}$$

The constant c in (2.1) is the *coupling coefficient* and $\mu \in \mathbb{R}$ is called the *Josephson current*. Meanwhile γ obviously plays the role of an external damping coefficient. The symbols $\dot{u}$ and $\ddot{u}$ denote the first- and the second-order derivatives of u with respect to time, respectively. In turn, the function ϕ may be assumed in general to be continuously differentiable over $(0, +\infty)$, though for practical purposes we may think of it as

$$\phi(t) = A \sin(\Omega t), \quad \forall t > 0. \tag{2.3}$$

Here A is a positive number and $0 < \Omega < 1$. Note that the Hamiltonian of the nth node in the undamped regime is given by

$$H_n(t) = \frac{1}{2}\left[\dot{u}_n^2(t) + c^2 (u_{n+1}(t) - u_n(t))^2\right] + 1 - \cos u_n(t), \tag{2.4}$$

for each $n \in J_N$. Finally, the total energy of the system at each instant of time is provided by

$$E(t) = \sum_{n=1}^{N} H_n(t) + \frac{c^2}{2}(u_1(t) - u_0(t))^2. \tag{2.5}$$