Commun Nonlinear Sci Numer Simulat 56 (2018) 93–114

Contents lists available at ScienceDirect

Commun Nonlinear Sci Numer Simulat

journal homepage: www.elsevier.com/locate/cnsns

Research paper

Genealogy and stability of periodic orbit families around uniformly rotating asteroids

Xiyun Hou*, Xiaosheng Xin, Jinglang Feng

School of Astronomy and Space Science, Nanjing University, 210093, China

ARTICLE INFO

Article history:
Received 9 August 2016
Revised 8 June 2017
Accepted 5 July 2017
Available online 8 July 2017

Keywords:
Asteroid
Periodic orbit
Stability
Resonance

ABSTRACT

Resonance orbits around a uniformly rotating asteroid are studied from the approach of periodic orbits in this work. Three periodic families (denoted as I, II, and III in the paper) are fundamental in organizing the resonance families. For the planar case: (1) Genealogy and stability of Families I, II and the prograde resonance families are studied. For extremely irregular asteroids, family genealogy close to the asteroid is greatly distorted from that of the two body-problem (2BP), indicating that it is inappropriate to treat the orbital motions as perturbed Keplerian orbits. (2) Genealogy and stability of Family III are also studied. Stability of this family may be destroyed by the secular resonance between the orbital ascending node's precession and the asteroid's rotation. For the spatial case: (1) Genealogy of the near circular three-dimensional periodic families are studied. The genealogy may be broken apart by families of eccentric frozen orbits whose argument of perigee is "frozen" in space. (2) The joint effects between the secular resonance and the orbital resonances may cause instability to three-dimensional orbital motion with orbit inclinations close to the critical values. Applying the general methodology to a case study — the asteroid Eros and also considering higher order non-spherical terms, some extraordinary orbits are found, such as the ones with orbital plane co-rotating with the asteroid, and the stable frozen orbits with argument of perigee librating around values different from 0°, 90°, 180°, 270°.

1. Introduction

One approach to study the orbital dynamics close to a uniformly rotating irregular asteroid is through the periodic orbits in the asteroid's body-fixed frame (BFF). There have been some papers on this problem [15–17,20,21,26,30–33,38,39]. These studies generally focus on computing specific periodic families and their stability properties, with little or no intention to study their inherent relations. Some previous papers [16,17] did notice the existence of inherent relationship between these families, but the relationship is not systematically investigated. Different from these works, in this contribution, we focus on the relationship (we use the terminology "genealogy") between periodic families and how the genealogy changes with the perturbations. The method in this work — first compute periodic families in the 2BP and then continue them to the perturbed case — is traditional in celestial mechanics and is extensively used in dynamical systems such as the circular restricted three-body problem [10]. The method itself is not new, but is rarely adopted to treat orbital motions with non-spherical perturbations. Through this method, the main purpose of the current study is not only to find stable periodic orbits

* Corresponding author.
E-mail address: silence@nju.edu.cn (X. Hou).

http://dx.doi.org/10.1016/j.cnsns.2017.07.004
1007-5704/

in the presence of non-spherical perturbations, but also to provide answers to the following question: How do the genealogy and stability of periodic families of the 2BP change with non-spherical perturbations? Three families are fundamental in answering this question. Their members are circular orbits in the unperturbed 2BP and remain near-circular in the presence of perturbations. We denote them respectively as I, II, and III. Studies show:

1. For the planar case, in the presence of non-spherical perturbations, Family I (II) breaks up into infinite pieces at the 1st order inner (outer) resonances. If the non-spherical terms are extremely large, genealogy of periodic families close to the asteroid is completely different from that of the 2BP, indicating that it is inappropriate anymore to describe these orbital motions as perturbed Keplerian orbits. On the other hand, genealogy of Family III is not affected by the 1st order resonances, but may also be broken apart by the secular resonances.
2. In the absence of non-spherical perturbations, the three-dimensional (3D) resonance periodic orbits play the role of "bridges" connecting the bifurcation orbits of Family III with those of Family I (II). In the presence of non-spherical perturbations, this genealogy may be broken apart by families of eccentric frozen orbits whose argument of perigee is "frozen" in space. Moreover, the joint effects between the orbital resonances and the secular resonance destroy the stability of orbital motions with orbit inclinations close to the critical values.

In this work, most of the studies are carried out in the 2nd order and 2nd degree (2OD) gravity, but at the end of the work, the asteroid Eros is taken as an example to show the effects of high-order non-spherical terms. Some extraordinary orbits are found, such as orbits with orbital plane co-rotating with the asteroid, and stable frozen orbits with their argument of perigee librating around values different from 0°, 90°, 180°, 270°.

2. Equations of motion

In the BFF of a uniformly rotating asteroid, the orbit of a small body follows

$$\begin{cases} \ddot{x} - 2n_a\dot{y} - n_a^2 x = \frac{\partial V}{\partial x} \\ \ddot{y} + 2n_a\dot{x} - n_a^2 y = \frac{\partial V}{\partial y} \\ \ddot{z} = \frac{\partial V}{\partial z} \end{cases} \tag{1}$$

where n_a is the asteroid's rotation speed. V is the minus of the asteroid's potential. When expressed with spherical harmonics, it has the following form [18]

$$V = V_0 + \Delta V = \frac{Gm_a}{r} + \frac{Gm_a}{r}\sum_{l=1}^{\infty}\sum_{m=0}^{l}\left(\frac{a_c}{r}\right)^l \bar{P}_{lm}(\sin\phi)[\bar{C}_{lm}\cos m\theta + \bar{S}\sin m\theta] \tag{2}$$

where $\bar{C}_{lm}, \bar{S}_{lm}$ are normalized Stokes coefficients, and $\bar{P}_{lm}$ is normalized associated Legendre function. m_a is the mass of the asteroid. ϕ is the latitude, and θ is the longitude of the massless particle in the BFF of the asteroid. Eq. (1) allows an integral

$$n_a^2(x^2 + y^2) + 2V - (\dot{x}^2 + \dot{y}^2 + \dot{z}^2) = C \tag{3}$$

This integral has a same form as the Jacobi integral of the circular restricted 3-body problem, so we also call it Jacobi integral and C as the Jacobi constant. Assuming the asteroid as a particle, the radius of the synchronous orbit is

$$r_{syn} = \left(\frac{Gm_aT_a^2}{4\pi^2}\right)^{\frac{1}{3}} \tag{4}$$

where $T_a = 2\pi/n_a$ is the rotation period of the asteroid. Taking following units in our work

$$[L] = r_{syn}, \quad [M] = m_a, \quad [T] = \sqrt{[L]^3/G[M]} \tag{5}$$

It's easy to show that the rotation speed of the asteroid equals 1 when above units are used. The first order partial derivatives of V with respect to the coordinates which appears in Eq. (1) are given in the appendix, along with the second order partial derivatives which are necessary when computing the Monodromy matrix of periodic orbits.

Different asteroids have different shapes and mass distributions, and thus have different gravity fields. Except for some special cases [22], the dominant non-spherical terms are J_2 and J_{22} (i.e., the 2OD gravity). It is difficult to get some common knowledge on diverse gravity fields, so we first carry out the study in the 2OD gravity and then discuss the effects of higher order non-spherical terms for specific asteroids. To separate the 2OD terms from the higher order terms in the gravity field, we didn't use the more accurate but more sophisticated polyhedron model [36]. Nevertheless, the way to compute periodic families (see Section 3) also applies when the polyhedron model is used. To give physical interpretations of the J_2, J_{22} values used in our work, we use the tri-axial ellipsoidal shape model for the asteroid. Suppose the semi axes of the ellipsoid are $a \geq b \geq c$, we have [1]

$$J_2 = \frac{a^2 + b^2 - 2c^2}{10a_{ref}^2} = \frac{1}{10}\left(\frac{n_a^2}{G\rho}\right)^{2/3}\frac{1 + \alpha^2 - 2\beta^2}{(\alpha\beta)^{2/3}}$$