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Research paper

Double-well chimeras in 2D lattice of chaotic bistable elements

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ABSTRACT

We investigate spatio-temporal dynamics of a 2D ensemble of nonlocally coupled chaotic cubic maps in a bistability regime. In particular, we perform a detailed study on the transition "coherence – incoherence" for varying coupling strength for a fixed interaction radius. For the 2D ensemble we show the appearance of amplitude and phase chimera states previously reported for 1D ensembles of nonlocally coupled chaotic systems. Moreover, we uncover a novel type of chimera state, *double-well chimera*, which occurs due to the interplay of the bistability of the local dynamics and the 2D ensemble structure. Additionally, we find double-well chimera behavior for steady states which we call *double-well chimera death*. A distinguishing feature of chimera patterns observed in the lattice is that they mainly combine clusters of different chimera types: phase, amplitude and double-well chimeras.

1. Introduction

The study of the dynamics of multicomponent systems, such as nonlinear networks, ensembles of interacting nonlinear oscillators and distributed in space active systems is one of the most important directions in nonlinear dynamics. Interaction of nonlinear elements constituting complex systems results in a great variety of dynamical regimes and spatial structures. These questions are covered in monographs [1–6] and in many articles (for example, [7–13]). In these and other works it is shown that one of the main features of nonlinear networks and spatially-organized active systems is the formation of patterns, such as synchronization clusters, spatial intermittency, steady state patterns, spatial chaos, various types of regular and chaotic wave processes, for example, spiral waves.

A new type of structures has been recently found: chimera states [5,14–17]. This structure is especially typical for ensembles of oscillators with nonlocal interactions. Chimera is a partial synchronization pattern consisting of coherent and incoherent clusters. Elements from the coherent cluster act in synchrony, while the oscillators from the incoherent cluster are not correlated and form a domain of spatial chaos with fixed borders. Chimeras were found in ensembles of phase oscillators [18–22], periodic self-sustained oscillators [23–28], chaotic oscillators and chaotic return maps [29–33], networks of oscillatory elements containing blocks of excitable elements [34] or only excitable units [35]. Chimera structures were obtained not only in numerical simulations but also in experiments [36–39]. Besides systems with nonlocal coupling chimera

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states were found for the case of global coupling [21,25,39,40], local coupling [41–43] and also in single oscillators with delayed feedback where the delay time plays the role of virtual-space [44,45].

Chimeras in ensembles of chaotic oscillators significantly differ from chimera states in ensembles of phase oscillators and periodic self-sustained oscillators. Their emergence does not occur due to detuning of mean frequency of oscillators in incoherent clusters (which is typical for phase oscillators). Not any kind of chaotic behavior promotes chimera states. Apparently, hyperbolic chaos impedes the occurrence of chimeras [30]. The models of chaotic systems (such as logistic map, Rössler oscillator) demonstrating chimera states in networks of interacting units [29–32] are characterized by the regime of nonhyperbolic chaos, which occurs due to a cascade of period doubling bifurcations (Feigenbaum scenario [46]). The dynamics of ensembles of such elements with local interactions (for example, [7,9,47,48]) shows a high level of multistability with a variety of coexisting states in the case of weak coupling. One can assume that nonlocal coupling makes chimera states more favorable. Therefore, in the present work we also focus on the nonlocal type of the interaction between the network elements.

Chaotic dynamics of a nonhyperbolic type is not restricted to the models of chaotic oscillators mentioned above. The example of the bistable system in radioelectronics is the Chua circuit [49]. For changing parameters it demonstrates the bistability of equilibrium points, limit circle and chaotic attractors. The simplest analogy of the Chua circuit with the same regimes is a cubic map [48,50]. On the one hand, chaos in the cubic map is born through the Feigenbaum scenario as it is also the case for a logistic map. Therefore, one can assume that chimera states found in the ensembles of logistic maps with nonlocal coupling, should also be observed for networks of cubic maps. On the other hand, chaotic bistability and merging bifurcation of chaotic attractors may introduce new features and lead to a novel type of chimera behavior. These types of the chimera structures were not almost considered. An exception is the paper by Maistrenko et al. [51] where elements with strong asymmetric bistability were used to form chimera structures. One of the bistable state is a chaotic attractor and the other one is a stable equilibrium point. The clusters consisting of the interacting elements from these both states are formed. Consequently, they are either coherent or incoherent. However, the entire diversity of chimera structures, which is a consequence of bistability, is not described in [51]. We show another type of the chimera states, describe their evolution with parameter variations. In this work we aim to investigate the patterns, and in particular chimera structures, occurring in networks of cubic maps and uncover their characteristic features related to the bistability of the chaotic behavior.

Another question we address here is the impact of network topology on the occurrence of chimera patterns. Chimeras have been found for one-dimensional spatially-organized oscillatory ensembles with periodic boundary conditions (a ring) with nonlocal interaction, i.e. each element is coupled to a certain number of its nearest neighbors. There are also works on 2D-lattice [19,36,39] and 3D-lattice [22]. The chimeras in the form of spiral waves [52] have been observed in the model of 2D medium with phase dynamics of elements. In the works on chimeras in multidimensional ensembles a phase oscillator or its analog with discrete time [36] is used for the local dynamics. The occurrence of chimera patterns in 2D ensembles of elements with chaotic behavior remains to be understood.

In the present work we investigate two-dimensional lattice of cubic maps with periodic boundary conditions and non-local interaction between the elements. The main question we address here is how the interplay of the bistability of the local dynamics and the 2D network structure influences the transition from coherence to incoherence. Moreover, we analyze the impact of the bifurcation of merging of chaotic attractors, which occurs in a single element, on the behavior of the network. In particular, we study the appearance of chimera patterns and uncover a novel type of chimera state which we call *double-well chimera*.

2. Model

We study a 2D square lattice of nonlocally coupled cubic maps which is described by the following system of equations (1):

$$\begin{aligned} x_{i,j}(n+1) &= f(x_{i,j}(n)) + \frac{\sigma}{B} \sum_{\substack{k=i-R\\ p=j-R}}^{\substack{i+R\\ j+R}} \big(f(x_{k,p}(n)) - f(x_{i,j}(n))\big), \quad i,j = 1,\dots N, \\ f(x) &= \big(\alpha x - x^3\big)\exp\left[-\frac{x^2}{\beta}\right], \quad B = (1+2R)^2 - 1, \\ x_{i+N,j}(n) &= x_{i,j}(n), \quad x_{i,j+N}(n) = x_{i,j}(n), \end{aligned} \tag{1}$$

where i and j specify the position of a lattice element and can be considered as discrete spatial coordinates (X and Y), $n = 0, 1, 2, \dots$ determines time, $f(x)$ defines the local dynamics and depends on the parameters α and β. The size of system under study is N^2, where N is the number of elements in a one direction (along the X or Y axis). Boundary conditions are periodic in both directions. The interaction between the nodes of the network is nonlocal: each element of the lattice is coupled to its nearest neighbors located inside the square with the edge $2R+1$ and the element being located in the center of this square (see Fig. 1). Thus, $B = (2R+1)^2 - 1$ is the total number of links for each element. Coupling strength is characterized by the parameter σ. For multidimensional ensembles a sphere with a specified radius R is often considered instead of the square [19,22,36]. In this case R is normalized by the total number of elements and is called coupling radius.