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Research paper

Dynamic intersectoral models with power-law memory

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ABSTRACT

Intersectoral dynamic models with power-law memory are proposed. The equations of open and closed intersectoral models, in which the memory effects are described by the Caputo derivatives of non-integer orders, are derived. We suggest solutions of these equations, which have the form of linear combinations of the Mittag-Leffler functions and which are characterized by different effective growth rates. Examples of intersectoral dynamics with power-law memory are suggested for two sectoral cases. We formulate two principles of intersectoral dynamics with memory: the principle of changing of technological growth rates and the principle of domination change. It has been shown that in the input–output economic dynamics the effects of fading memory can change the economic growth rate and dominant behavior of economic sectors.

1. Introduction

Dynamic models describe the dynamics of intersectoral balance of the gross product and the final product (national income) in the economy. These models are based on the equations of input-output balance in monetary terms that describe the production and distribution of the gross and final products between sectors. It takes into account the intersectoral production links, the use of material resources, the production and distribution of the national income. In the balance equation each sector is considered twice as a consumer and as a producer. This leads us to matrix form of the balance equations. A distinguishing feature of the intersectoral model is a description of the "input-output" balance equations in the matrix form. The matrix equation of intersectoral balance assumes that each product has only one production (one sector), and each production (industry) produces only one type of product. In the dynamic intersectoral models, the exogenous and endogenous variables are described by matrices.

One of the most famous models is the dynamic intersectoral model developed by Nobel laureate Wassily W. Leontief. The dynamic intersectoral model has been proposed by Leontief in the fifties of the XX century [1–4]. The Leontief dynamic model is an economic model of growth of gross national product and national income.

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The dynamic intersectoral models use different assumptions and are not taken into account some of the economic factors. One of the assumptions, which are commonly used in dynamic intersectoral models, is the neglect of the memory effects. In fact, the dynamic intersectoral models assumed that economic agents cannot remember history of changes of the endogenous and exogenous variables. As a result, we can say that these models describe only processes, in which all agents have full amnesia. The memory effects can play an important role in economics and natural sciences [5–9].

In macroeconomics, the memory can be considered as a property of economic processes, which characterizes the dependence of this process at a given time on the states in the past. In economic process with memory the endogenous and exogenous variables at a given time depend on their values at previous instants of time. In macroeconomic processes, the behavior of economic agents based not only on information on the state of the process $\{t, X(t)\}$ at a given moment of time t, but also on the use of information about the states $\{\tau, X(\tau)\}$ at time instants $\tau \in [0, t]$. A presence of memory in the processes means that there is an endogenous variable that depends not only on values of an exogenous variable at present time, but also on its values at previous instants of time. A memory effect is related with the fact that the same change of the exogenous variable can leads to the different change of the corresponding endogenous variable. This leads us to the multivalued dependencies of these variables. The multivalued dependencies are caused by the fact that the economic agents remember previous changes of these variables, and therefore can react differently. As a result, an identical change of the exogenous variable may lead to the different dynamics of endogenous variables.

In economics, the concept of memory can be considered by analogy with fractional dynamics [8, p. 394–396]. In this paper, we propose a method of accounting the power-law memory in the construction of dynamic intersectoral models in the form of generalization of the dynamic Leontief model with continuous time. As a mathematical tool we use the theory of differential equations with derivatives of non-integer order [10–14]. Our consideration is based on the concept of the accelerator with memory and marginal value of non-integer order, which are suggested in [15–19]. This paper presents and analyzes the solutions of the equations of the closed and open intersectoral dynamic models by using the solutions on fractional differential equations, which are considered in [12,13,20–22].

2. Dynamic intersectoral model without memory

Let us consider intersectoral model, where we assume that n kinds of products are produced and used. Each sector produces only one type of products, and each product is produced in a certain sector. Characteristics of production processes are assumed to be constant, that is, we will not take into account that technological progress can lead to changes in production technology. In addition, the import of goods and materials and the use of non-renewable resources will not be considered in the model. Dynamic intersectoral model will be formulated in the framework of continuous time approach.

Let us describe a derivation of input-output balance of equation of dynamic Leontief model. In the Leontief model, the gross product (gross output) is described by the vector $X(t) = (X_k(t))$, and it is divided into two parts

$$X(t) = Z(t) + Y(t), \tag{1}$$

where $Y(t) = (Y_k(t))$ is a vector of the final product; $Z(t) = (Z_k(t))$ is the vector of the intermediate product, where k= 1,…,n are production sectors. The final product is distributed to the investments and the non-productive consumption

$$Y(t) = I(t) + C(t), \tag{2}$$

where $I(t) = (I_k(t))$ is a vector of investments; $C(t) = (C_k(t))$ is the vector of products of nonproductive consumption (including non-productive accumulation), where k = 1,…, n are production sectors. The Leontief dynamic model assumes the performance of the balance Eqs. (1) and (2) for any $t > 0$. These equations describe the dynamic equilibrium of the economy as a whole. For this reason, the dynamic Leontief model is a dynamic model of the “input-output” balance. Substituting the expression of the final product (2) into formula (1), we obtain the balance equation

$$X(t) = Z(t) + I(t) + C(t). \tag{3}$$

To get the Leontief model equation from the balance Eq. (3), it is necessary to eliminate the endogenous (internal) variables Z(t) and I(t). To do this, we should give dependence of Z(t) and I(t) on the exogenous variable X(t).

The Leontief model assumes constancy of coefficients of direct material costs of production. The dependence of the intermediate product on the gross product is assumed in the form of direct proportionality. This allows us to express the vector of intermediate products Z(t) through the multiplication of the matrix of direct material costs A and the vector of gross product X(t) in the form of the matrix multiplier equation

$$Z(t) = A \cdot X(t), \tag{4}$$

where $A = (a_{ij})$ is the square matrix of n-th order with coefficients a_{ij}, which describe the direct material costs of *i*th sector (i= 1,…,n) in the production of a unit of output *j*th sector (j= 1,…,n). The matrix A is assumed to be constant, that is, it does not change with time. In the dynamic Leontief model the coefficients a_{ij} include not only the direct material costs, but also the costs of disposal compensation and repair the basic production assets. Therefore, the elements of the main diagonal of the matrix A are nonzero.

The dynamic Leontief model is based on the assumption of the relationship between the accumulation and of the growth of gross output. This relationship is implemented by using matrix of capital intensity of production growth. In addition, it is