



Research paper

A modified Leslie–Gower predator–prey interaction model and parameter identifiability

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ABSTRACT

In this work, bifurcation and a systematic approach for estimation of identifiable parameters of a modified Leslie–Gower predator–prey system with Crowley–Martin functional response and prey refuge is discussed. Global asymptotic stability is discussed by applying fluctuation lemma. The system undergoes into Hopf bifurcation with respect to parameters intrinsic growth rate of predators (s) and prey reserve (m). The stability of Hopf bifurcation is also discussed by calculating Lyapunov number. The sensitivity analysis of the considered model system with respect to all variables is performed which also supports our theoretical study. To estimate the unknown parameter from the data, an optimization procedure (pseudo-random search algorithm) is adopted. System responses and phase plots for estimated parameters are also compared with true noise free data. It is found that the system dynamics with true set of parametric values is similar to the estimated parametric values. Numerical simulations are presented to substantiate the analytical findings.

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1. Introduction

Predator–prey model [1,2] is one of the dominant theme in both ecology and mathematical ecology due to its universal existence and importance with many concerned biological systems. Let $x(t)$ and $y(t)$ are prey and predator population densities at time t , respectively. The classical Gause-type predator–prey model [3,4] system with logistic growth rate of prey species is given by

$$\frac{dx}{dt} = rx \left(1 - \frac{x}{K} \right) - f(x)y, \quad \frac{dy}{dt} = y(-\alpha + \beta g(x)), \quad (1)$$

subjected to the positive initial conditions $x(0) > 0$, $y(0) > 0$, which are biologically meaningful. The parameters r and α denote the intrinsic growth rate of prey and natural mortality rate of predator, respectively. The value K is the carrying capacity for prey. The function $f(x)$ stands for feeding rate of an average predator per unit time i.e., functional response of predators to the prey (refer [5–16] and references cited therein).

In ecology, a barrier for population dispersal (separation of populations) can be termed as “refuge” (see [28–30]). There has been great deal of research on the effect of reserve zone/refuge in the dynamics of prey–predator model (see in detail in [31–36]). Most of researchers have shown that refugia have a stabilizing effect on prey–predator model. See [33,34] and

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references therein for the stability of prey-predator model incorporating prey refuge. A prey-predator resource model is explained with optimum harvesting policy in [35,36]. In [37], author discussed the stability analysis of prey-predator model with Holling type II functional response in a two patch environment: one accessible to both prey and predator (patch 1) and the other one being a refuge for the prey (patch 2). Chen et al. [38] discussed a Leslie–Gower predator-prey model incorporating a prey refuge. Recently Tripathi et al. [49] presented complete dynamical analysis of a prey-predator model with Beddington–DeAngelis type function response [17,18,32] incorporating a prey refuge. Hoy [39] mentioned that “hotspots” of high spider mite densities in almond orchards can trigger orchard-wide outbreaks. These hotspots are areas in which the predator is not successfully controlling the prey and therefore can be considered refugia. Hence in these areas predator species no longer exist.

Here we consider that in the absence of predator population, prey species follow logistic growth rate. The growth of the predator population is of logistic form where the carrying capacity of the predator environment is proportional to the number of prey (prey dependent carrying capacity for the predator) [25–27]. It is assumed that in case of severe scarcity, the predator (y) can switch over the other population. Further, it is also assumed that the refuge protecting mx of prey, where $m \in [0, 1]$, is constant and hence $(1 - m)x$ is only prey available to predator. Under the above assumptions, the model system with Crowley–Martin functional response ([10–22]) and prey refuge to be investigated is:

$$\frac{dx}{dt} = rx \left(1 - \frac{x}{K}\right) - \frac{\mu(1 - m)xy}{a + b(1 - m)x + cy + d(1 - m)xy}, \quad \frac{dy}{dt} = sy \left(1 - \frac{vy}{(1 - m)x + n}\right). \quad (2)$$

Here, all the parameters $r, s, K, \mu, v, a, b, c, d, n$ in the model system (2), assume only positive values and will be considered as constants throughout our discussion. The parameters μ, b, c, d are also termed as saturating Crowley–Martin type functional response parameters.

Models imitating complex physical phenomena usually involves more than one parameters and its applicability requires estimation of these parameters. It is well known that the parameters are estimated from the provided noisy observations of physical phenomena under consideration. This inverse process of estimating parameters can be modelled as minimizing non-linear least square problem. Hence, an optimization procedure can be employed to estimate these hidden parameter. For example, recently Tripathi et al. [49] used twin experiment [42,43] for parameter estimation. In the same study, authors used LX-PM genetic algorithm [44,45] to estimate single parameter of considered prey-predator model. However, if number of parameters to be estimated are more than one, then functional relation between the parameters can make the system non-identifiable. Non-identifiability can also arise due to uncertainty in noisy data. Hence, it is important to check system identifiability before estimating parameters of the models.

A systematic approach for model identifiability and estimation of identifiable parameters for models involving system of differential equations was developed by Brun et al. [46]. Since then similar approach was adopted in several studies relating lake modelling [52], river modelling [53], material science [54], surface hydrology modelling [55], air pollution modelling [51] etc.

In the present paper, our main purpose is the mathematical analysis and estimation of identifiable parameters of the model system (3). Estimation of identifiable parameters are discussed by assessing the local sensitivity system response with respect to model parameters. We wish to show that the sensitivity analysis of model parameters along with estimation of identifiable parameters of the system supports the theoretical findings.

The rest of this paper is organized as follows. In the next section, we discuss the mathematical results associated to the system (3). In Section 4, we also perform some carefully designed numerical simulations to validate our analytical findings. Parameter estimation with sensitivity and identifiability, numerical algorithm adopted are discussed in Section 5. We also solve the inverse problem of estimation of identifiable model parameters using sampled data of the system. Paper ends with concluding remarks and ecological significance of the analytical and numerical findings.

2. Mathematical analysis

To reduce the arising complexity in the dynamical analysis and interpretation of results, we non-dimensionalize the model system (2) with the following set of variables and parameters:

$$\bar{t} = rt, \quad \bar{x} = \frac{x}{K}, \quad \frac{\mu}{a}y = \bar{y}, \quad \frac{bK}{a} = \bar{b}, \quad \frac{c}{\mu} = \bar{c}, \quad \frac{dK}{\mu} = \bar{d}, \quad \frac{s}{dr} = \bar{s}, \quad \frac{av}{\mu K} = \bar{v}, \quad \frac{n}{K} = \bar{n}.$$

Making these changes and dropping the bars from the resulting equations yield the following systems without r, μ and K :

$$\begin{aligned} \frac{dx}{dt} &= x \left(1 - x - \frac{(1 - m)y}{1 + b(1 - m)x + cy + d(1 - m)xy}\right) \equiv x f^{(1)}(x, y), \\ \frac{dy}{dt} &= sy \left(1 - \frac{vy}{(1 - m)x + n}\right) \equiv y f^{(2)}(x, y), \end{aligned} \quad (3)$$

with the initial conditions

$$x(0) = x_0 > 0, \quad y(0) = y_0 > 0. \quad (4)$$

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