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Research paper

Chaos control in delayed phase space constructed by the Takens embedding theory

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A B S T R A C T

In this paper, the problem of chaos control in discrete-time chaotic systems with unknown governing equations and limited measurable states is investigated. Using the time-series of only one measurable state, an algorithm is proposed to stabilize unstable fixed points. The approach consists of three steps: first, using Takens embedding theory, a delayed phase space preserving the topological characteristics of the unknown system is reconstructed. Second, a dynamic model is identified by recursive least squares method to estimate the time-series data in the delayed phase space. Finally, based on the reconstructed model, an appropriate linear delayed feedback controller is obtained for stabilizing unstable fixed points of the system. Controller gains are computed using a systematic approach. The effectiveness of the proposed algorithm is examined by applying it to the generalized hyper-chaotic Henon system, prey-predator population map, and the discrete-time Lorenz system.

1. Introduction

The last two decades have witnessed a great deal of interest in nonlinear dynamical systems in general and chaotic nonlinear systems in particular [1,2]. In fact, the emergence of new mathematical and numerical tools has played a crucial role in understanding and describing the concept of chaos. The research on chaotic systems has been done from a number of different standpoints, among which chaos control is the main purpose of our present work.

The pioneering work on chaos control introduced by Ott, Grebogi, and York, stimulated scientists to find innovative methods for the control of chaotic systems [3]. One of the most appealing methods which can be implemented to most of chaotic systems was suggested by Pyragas [4]. One advantage of his work is that the presented method does not need any information about periodic solutions other than their period. The Pyragas delayed feedback control has been applied to discrete-time chaotic systems [5,6] as well as continuous-time chaotic systems [7]. So many other methods for stabilizing chaotic systems have been suggested afterward such as: feedback linearization [8,9], variable structure control [10], sliding mode control [11–13], backstepping [14], and fuzzy control [15,16], to name but a few.

In the present paper, we aim at chaos control in discrete-time chaotic systems with unknown governing equations and limited measurable states using Takens' embedding theory [17]. Takens' theory shows how a time-series of measurements of a single observable state can be often used to reconstruct qualitative features of the phase space of the system. The technique described by Takens, named the method of delays, is simple such that it can be applied to each time-series.

Generally, chaos control methods are divided into two groups. The first group contains model-based control methods, which use the governing equations of the system to obtain the control actions. These methods cannot be utilized when

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the governing equations of the system is unknown. The second group contains model-free control methods such as the Pyragas method, which is a delayed feedback control method. In the Pyragas method the controller gains are obtained using a time-consuming trial and error process. In this paper, a linear delayed feedback control method is used to control chaos in the systems. A systematic algorithm is proposed to obtain the controller gains, which is more valuable compared with the time-consuming trial and error process.

This paper is organized as follows. In the second section, the problem is described and made clear. In the third section, the developed algorithm is described. In the fourth section, the proposed algorithm is applied to the generalized hyper-chaotic Henon system, prey-predator population map, and the discrete-time Lorenz system. Its effectiveness is illustrated using numerical simulations. In the last section, some concluding remarks are presented.

2. Problem statement

Consider the following general n-dimensional discrete-time chaotic system.

$$\mathbf{x}_{k+1} = \mathbf{F}(\mathbf{x}_k), \quad \mathbf{x}_k = [x_{1\,k}, x_{2\,k}, \dots, x_{n\,k}]^T \tag{1}$$

Suppose that the system has some un-measurable states, and no information about the function **F** is known. Therefore, the only available information about the system is the time-series of the measurable states. The objective is to control chaos by stabilizing UFPs of the chaotic system. Here it is assumed that the period of the UFP is one, without loss of generality of the problem. Using the concept of Takens embedding theory, Time-series of one measurable state is used to establish an algorithm accomplishing the objective.

3. Description of the algorithm

The proposed algorithm consists of three steps described in the following sections.

3.1. Reconstructing a delayed phase space

Based on the Takens embedding theory, a new phase space can be derived from one measurable state of the system such that the topological characteristics of the system are reserved. The original phase space is denoted by **x**, and the reconstructed phase space is denoted by **y**, which is constructed as follows:

$$\mathbf{x} : \left\{\begin{matrix} x_1 \\ x_2 \\ x_3 \\ . \\ . \\ . \\ x_n \end{matrix}\right\} \xrightarrow{\varphi} \mathbf{y} : \left\{\begin{matrix} x_{1\,k} \\ x_{1\,k-1} \\ x_{1\,k-2} \\ . \\ . \\ . \\ x_{1\,k-(m-1)} \end{matrix}\right\} \tag{2}$$

where φ is a smooth diffeomorphism map, and m is the dimension of reconstructed phase space. Takens theory states that $m \geq 2d + 1$ where d is the capacity dimension or correlation dimension of the trajectory of the system [18]. The proposed dimension for m by Takens is usually more than enough. In other words, the minimum value $2d + 1$ for m is a conservative amount, and m can often be assumed less than $2d + 1$. The False nearest neighbors algorithm is a well-known method for obtaining the proper value of m [19]. This method was developed based on the fact that choosing too low embedding dimension results in points that are far apart in the original phase space being moved closer together in the reconstruction space. In [20], a more practical method for determining the minimum embedding dimension is introduced. This method is used in determining minimum appropriate embedding dimension.

3.2. Modeling

Now that the reconstructed phase space is available, the algorithm is followed by finding a proper dynamic model. There is many dynamic models which have been employed for various problems, namely global polynomials [21,22], local polynomials [23], neural networks [23], radial basis networks [24], and local linear fits [23]. In most cases among the known discrete-time chaotic systems, the functionality of the system consists of polynomial functions. Also, global polynomials fulfill the required nonlinearity of the system. Therefore, global polynomials are selected here for modeling.

Global polynomials are linear combination of polynomial functions. Each polynomial is named a basis function, and the whole polynomials are named a dictionary of basis functions. To find correct basis functions for the dynamic model, dictionary of basis functions is started by making polynomials with maximum exponent of 2 from the states of the reconstructed phase space. Using recursive least squares (RLS) method [25], converged coefficients of each basis function and estimation error are computed. If the estimation error is not low enough, by increasing the maximum exponent of polynomials, more candidates of basis functions are incorporated into the dictionary. By taking y_k as the measurable state, the reconstructed system is defined as:

$$y_{k+1} = f(\mathbf{y}_k) \qquad \mathbf{y}_k = [y_k, y_{k-1}, \dots, y_{k-m+1}]^T \tag{3}$$