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Research paper

Negative effective mass in acoustic metamaterial with nonlinear mass-in-mass subsystems

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ABSTRACT

In this paper the dynamics of the nonlinear mass-in-mass system as the basic subsystem of the acoustic metamaterial is investigated. The excitation of the system is in the form of the Jacobi elliptic function. The corresponding model to this forcing is the mass-in-mass system with cubic nonlinearity of the Duffing type. Mathematical model of the motion is a system of two coupled strong nonlinear and nonhomogeneous second order differential equations. Particular solution to the system is obtained. The analytical solution of the problem is based on the simple and double integral of the cosine Jacobi function. In the paper the integrals are given in the form of series of trigonometric functions. These results are new one. After some modification the simplified solution in the first approximation is obtained. The result is convenient for discussion. Conditions for elimination of the motion of the mass 1 by connection of the nonlinear dynamic absorber (mass - spring system) are defined. In the consideration the effective mass ratio is introduced in the nonlinear mass-in-mass system. Negative effective mass ratio gives the absorption of vibrations with certain frequencies. The advantage of the nonlinear subunit in comparison to the linear one is that the frequency gap is significantly wider. Nevertheless, it has to be mentioned that the amplitude of vibration differs from zero for a small value. In the paper the analytical results are compared with numerical one and are in agreement.

1. Introduction

In the last ten years a significant attention is directed toward so called 'metamaterials' which have quite different properties than the natural materials and are suitable for elimination and absorption of the electromagnetic and acoustic waves. Usually, these materials are composites and their response on both electromagnetic and acoustic excitation is similar due to the analogy between electromagnetic and acoustic waves. Acoustic metamaterials are suitable for absorption of acoustic waves, while electromagnetic materials for reduction of electromagnetic waves. The basic element of acoustic metamaterials is usually modeled as mass-in-mass mechanical system (Fig. 1a): on the mass m_1 another mass m_2 is connected with a spring with rigidity coefficient is k_2. The excitation force acts on the mass m_1 and causes vibration. The mass-spring system has to absorb the critical frequency of the excited system by producing of the so called 'frequency gaps'. The values of parameters of the mass-spring system are determined to eliminate the motion of the mass m_1. In papers [1–3], mechanical properties of the linear mass-in-mass model are discussed. It is concluded that the undamped vibration absorber with

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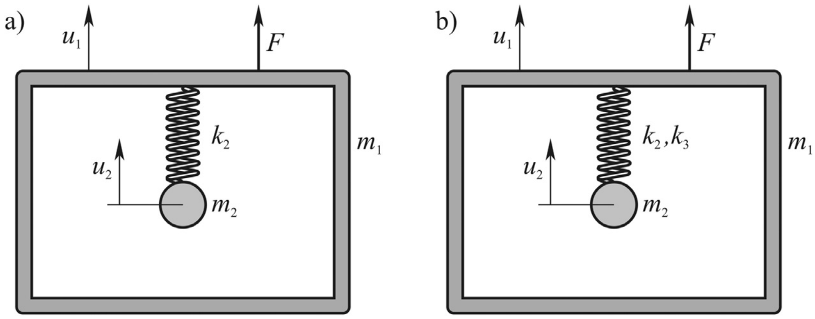

Fig. 1. Model of mass-in-mass system: a) linear, b)nonlinear.

frequency $\sqrt{k_2/m_2}$, which is equal to the frequency of the excitation, eliminates this certain frequency. In [4–7] various models of elastic metamaterials are presented in which the mass-in-mass subsystems are connected in specific ways. So, Milton and Willis [1] developed the theory of metastructure where the mass-in-mass systems are connected in a line while Wang et al. [7] considered the model with two mass-in-mass systems. In the papers the negative effective mass ratio [8–11] in metamaterials is introduced. Mass effective ratio is an artificial parameter which is calculated analogy as the parameters of electromagnetic metamaterials [12–14] and has not only positive but also negative values. It has to be mentioned that this parameter has not a physical sense and has nothing to do with real physical mass. Nevertheless, based on the sign of the effective mass the concept of optical and acoustical resonance is developed [15]. The special attention is given to influence of the negative effective mass on the properties of metamaterial. Besides, the considered subsystems of the metamaterial are assumed to be linear and the excitation has the form of a trigonometric function. The main disadvantage of the mentioned system is that the absorption frequency gap is narrow.

In this paper the new model of metamaterial subsystems is developed which available absorption of acoustic waves in much wider frequency region. The main property of the suggested model is its nonlinearity. The subsystem contains a mass m_1 on which the mass m_2 is connected with a spring with nonlinear properties. The excitation F is assumed in the form of the *cn* Jacobi elliptic function [16–20]

$$F = F_0 cn(\omega t, k^2), \tag{1}$$

where F_0 is the amplitude of excitation, ω is the frequency and k is the modulus of the Jacobi elliptic function. In the paper [21] the physical meaning of this type of excitation is discussed.

The paper is divided into five sections. After Introduction the model of nonlinear mass-in-mass system is developed. Mathematical description of the oscillation of the system is given. Analytical procedure for solving the system of two coupled nonlinear and nonhomogeneous differential equations is developed. As a result, amplitudes of vibration for both masses in the system are determined. The analytically obtained solutions are compared with numerical ones. In Section 3, conditions for the connected spring-mass system to be the dynamic undamped vibration absorber are obtained. We calculated the rigidity and mass parameters of the added system for which the motion of the mass m_1 is minimized. In Section 4, the effective mass ratio for the system with cubic nonlinearity is obtained. Conditions for acoustic resonance in the nonlinear system are determined. We obtained parameters for which the mass ratio is negative and the metamaterial has special absorption property. The paper ends with Conclusion. Finally, at the end of the paper, in Appendix, as an additional result, we give the expression of the integral $\int cn(\psi)d\psi$ i.e., as a special case of the function $\cos^{-1}(\mathrm{dn}u)$. Also the solution of the two times integration of the *cn* Jacobi elliptic function for the argument u, i.e., $\int(\int cn(\psi)d\psi)d\psi$ is calculated. The result of this integration was not previously mentioned in the literature.

2. Model of the nonlinear mass-in-mass system

In Fig. 1b the excited mass-in-mass system is shown. On the mass m_1, excited with (1), a spring-mass system is connected. For elimination of the influence of the excitation in the form of the *cn* Jacobi elliptic function we suggest to connect the mass with a spring with strong cubic nonlinearity. Namely, the elastic force is a nonlinear deflection function. Mathematical model of the system is

$$m_1\ddot{u}_1 + k_2(u_1 - u_2) + k_3(u_1 - u_2)^3 = F_0 cn(\omega t, k^2), \tag{2}$$

$$m_2\ddot{u}_2 + k_2(u_2 - u_1) + k_3(u_2 - u_1)^3 = 0, \tag{3}$$

where k_2 and k_3 are rigidity coefficients of the linear and cubic terms of the elastic force of the spring, respectively. The model (3) is a system of two coupled strong nonlinear nonhomogeneous differential equations.