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Research paper

Investigation of stickiness influence in the anomalous transport and diffusion for a non-dissipative Fermi–Ulam model

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ABSTRACT

We study the dynamics of an ensemble of non interacting particles constrained by two infinitely heavy walls, where one of them is moving periodically in time, while the other is fixed. The system presents mixed dynamics, where the accessible region for the particle to diffuse chaotically is bordered by an invariant spanning curve. Statistical analysis for the root mean square velocity, considering high and low velocity ensembles, leads the dynamics to the same steady state plateau for long times. A transport investigation of the dynamics via escape basins reveals that depending of the initial velocity ensemble, the decay rates of the survival probability present different shapes and bumps, in a mix of exponential, power law and stretched exponential decays. After an analysis of step-size averages, we found that the stable manifolds play the role of a preferential path for faster escape, being responsible for the bumps and different shapes of the survival probability.

1. Introduction

Hamiltonian systems are typically non-integrable and non-ergodic [1–3], where their dynamics present mixed properties in the phase space, with KAM islands, invariant tori, spanning curves and chaotic seas. One of the main consequences of this mixed dynamics is the anomalous transport that a chaotic orbit may experience when passing near by stability regions. The orbit can stick to their boundaries, thus getting trapped around the bounded area of these islands and its cantori for a finite time (that could be long), in what it known as stickiness effect [4,5]. Applications of this trapping phenomenon can be found in many research areas as: fluid mechanics [6], plasma physics [7–9], celestial mechanics [10], acoustics [11], biology [12], among others (See Ref. [1,2] for reviews). This anomalous behavior serve as motivation for our study, where the interface between chaotic, quasi-periodic and stable dynamics is very complex and not yet fully understood and generates some open problems [13,14].

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A natural observable allowing the study of the statistical properties of the transport, in particular $\rho(n)$, the probability (given a suitable distribution of initial conditions) that an orbit does not escape through a hole until a time n. Here, the hole is defined as a predefined subset of the phase space. The most important aspect of this analysis is that the escape rate is very sensitive to the system dynamics. For strongly chaotic systems the decay is typically exponential [15–18], while systems that present mixed phase space the decay can be slower, presenting a mix of exponential with a power law [19–21], or even stretched exponential decay [22]. Indeed, when a non-exponential decay is observed the dynamics would require a long range correlation. This is a direct consequence of the stickiness influence in the dynamics.

The model under study in this paper is the Fermi–Ulam model (FUM). The FUM was proposed by Ulam in early60s [23] as an attempt to produce a prototype that could explain the Fermi Acceleration [24] (unbounded energy growth). The system consists of an ensemble of non interacting particles confined to move between two infinitely heavy walls, which the particles collide elastically. One wall is assumed to be fixed while the other one oscillates periodically in time. The phase space is mixed and contains periodic islands surrounded by a chaotic sea, which is limited by a set of invariant spanning curves [15,25]. This implies that we have a finite portion of the phase space for orbits to diffuse [15], which prevents the dynamics to exhibit unlimited diffusion in the velocity. The mechanics of the model leads to a complex variety of nonlinear phenomena in both conservative and dissipative dynamics [26–30]. From an experimental and quantum point of view, one can imagine the FUM as a schematic where an atom or a photon bounces under the influence of strong electromagnetic pulses, with applications in astrophysics [31], where the radiated energy represents a typical realization of an on-off intermittent process, atom-optics [32,33], quantum effects [34–36] and experimental devices [37,38], where atoms can be trapped in featured resonances by optical cavities and ultra cold potentials.

In this paper we investigate and seek to understand the stickiness influence in the transport for a non-dissipative FUM. Since the accessible phase space have a finite portion for the orbit to diffuse, it can be divided into two regions of high and low energy regimes, that depends on the initial velocity of the ensemble. In previous studies [15,25], only the lower ensemble of energy (basically composed by chaotic sea) was investigated, leaving aside the higher ensemble, which has more complicated dynamics with chains of islands, cantori and small portions of chaotic sea, and seems more interesting to be studied. So, in this paper we give focus to the higher ensemble, but not neglecting the lower one as well, yielding in a complete overview of the dynamical scenario for the FUM. Statistical analysis concerning the root mean square velocity shows that for both regimes we have a convergence to a steady state plateau for long time series. A transport analysis shows that there is stickiness in both ensembles, and it influences the decay rates of the survival probability, presenting different shapes and bumps in a mix of exponential, power law and stretched exponential decays. After an analysis of step-size averages, we found that the stable manifolds play the role of a preferential path for faster escape, being responsible for the bumps and different shapes of the survival probability. These results give support to the stickiness influence towards to the anomalous transport and diffusion, where orbits can produce an extreme slower decay rates of the survival probability, with different bumps and shapes when compared with regular chaotic motion, and also can be extended to other similar dynamical systems.

The paper is organized as follows: In Section 2 we describe the details of the FUM mapping and some chaotic properties. Section 3 is devoted to the statistical analysis of the average velocity, as well as, the investigation of the anomalous transport and diffusion, concerning escape basins, survival probability curves and histogram of frequencies. Finally, in Section 4 we drawn some final remarks, conclusions and perspectives.

2. The model, the mapping and chaotic properties

In this section we will describe the model under study, so called Fermi–Ulam model (FUM), which consists of the motion of a free particle that suffers elastic collisions with two heavy walls, where one of them is said to be fixed at $x = l$, and the other one is periodic oscillating around $x = 0$. Dissipation could be introduced in the system via inelastic collisions [39] where a damping coefficient can be considered on the walls [40]. Also, kinetic friction [41] and in flight dissipation [42] can be introduced as well. However, in this paper we will consider only the conservative version, where the collision with both walls are completely elastic. The dynamics of this system is described by a non-linear and measure preserving mapping for the variables velocity of the particle v and time t immediately after a nth collision of the particle with the moving wall.

There are two distinct versions of the dynamics description: the complete one, which consists in considering the complete movement of the time-dependent wall, and the simplified, that is often used to speed up numerical simulations, where the moving wall is set to be fixed, but the particle exchanges momentum and energy with it, as if the wall were normally moving. Both approaches produce a very similar dynamics considering conservative and dissipative cases [15,25]. We consider in this paper the complete version, whose the position of the vibrating wall is given by $x_w(t_n) = \varepsilon \cos wt_n$, where ε and w are respectively the amplitude and the frequency of oscillation.

The dynamics is described using a two-dimensional mapping, where the background formalism and mathematical tools backs to Pustyl′nikov [43]. We notice that are three control parameters, named ℓ, ε and w, and not all of them are relevant. We then define the following dimensionless and more convenient variables as: $V_n = v_n/wl$, $\epsilon = \varepsilon/l$ and measuring the time in terms of the number of oscillations of the moving wall $\phi_n = wt_n$. Starting with an initial condition (V_n, ϕ_n) with initial position of the particle given by $x_p(\phi_n) = \cos(\phi_n)$, the dynamics is evolved by a map T which gives the pair (V_{n+1}, ϕ_{n+1}) in