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Short communication

A fractional calculus perspective of distributed propeller design

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ABSTRACT

A new generation of aircraft with distributed propellers leads to operational performances superior to those exhibited by standard designs. Computational simulations and experimental tests show a reduction of fuel consumption and noise. This paper proposes an *analogy* between aerodynamics and electrical circuits. The model reveals properties similar to those of fractional-order systems and gives a deeper insight into the dynamics of multi-propeller coupling.

1. Introduction

Recent advances in aircraft propulsion opened up new designs with distributed actuators [1,2]. Key phrases such as “distributed electric propulsion”, “innovative propeller”, “electric propellers distributed along the leading edge of thin wings”,“multiple propeller array”, and others have been used to describe this novel and emerging concept. Aircraft design is moving from classical 2 or 3 propellers with metallic blades powered by piston engines or gas turbines, toward 4 or more electric motors driving carbon fiber blades (see diagram of Fig. 1). The new electric and wing assembly leads to less noise and pollution, and to more power efficient machines. In some cases it is also vertical takeoff and landing, but that is an independent characteristic. In this area we can mention [3] the NASA Pathfinder and Pathfinder Plus projects [4] followed by the NASA Centurion [5] and NASA Helios HP01 and HP03 aircraft [6] with 6, 8, 14, 14 and 10 engines, respectively. Other examples are the NASA X-57 Maxwell [7] and the Joby S2 [8] with 14 and 12 engines, intended to reduce fuel consumption and noise. Additional recent designs can be found in [9,10]. In all cases the new technology uses a multi-propeller distributed structure to obtain performances superior to those reached by means of conventional designs based on a small number of propellers.

In this paper we adopt a modeling based on the *analogy* between aerodynamics and electrical systems. The electrical *analog* reflects only a small part of the fluid, motor and aircraft systems, but reveals interesting effects representative of fractional dynamics. In fact, the distributed structure and the dynamical coupling between the multiple propellers leads to a model revealing a recursive placement of poles and zeros in the frequency domain typical of Fractional Calculus (FC) descriptions. This perspective is coherent with recent advances in circuit systems that propose models and structures inspired in FC [11–14].

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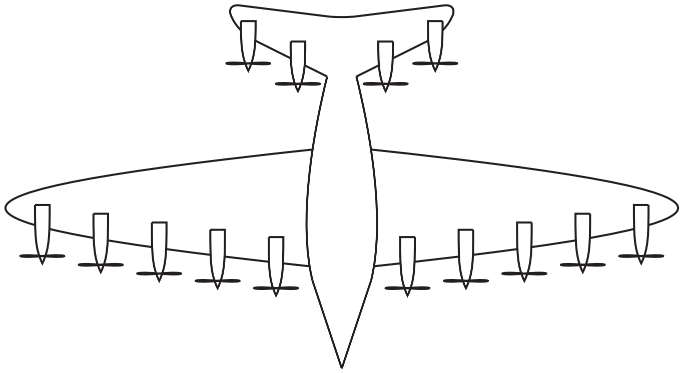

Fig. 1. Schematic diagram of aircraft with distributed propeller design.

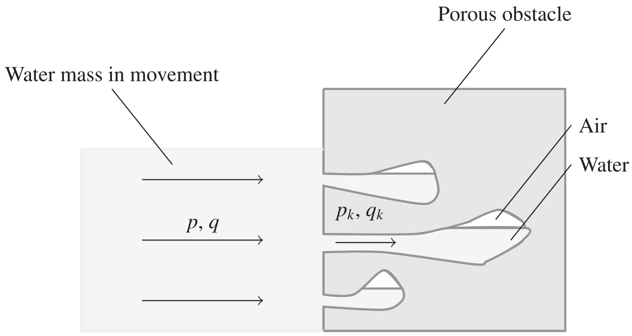

Fig. 2. Breakwater interface between liquid and solid materials [28–30].

In this line of thought the paper is organized as follows. Section 2 presents the fundamental concepts. Section 3 formulates the proposed analogy and studies the model properties. Finally, Section 4 summarizes the conclusions.

2. Fundamental concepts

FC generalizes [15–19] the concept of differentiation $D^\alpha f$ to orders $\alpha \in \mathbb{R}$. During the last years FC attracted considerable attention from the research community since it provides a valuable tool for modeling dynamical systems with long range memory effects [20–27]. Using the Fourier transform and for zero initial conditions we have the relationship:

$$\mathcal{F}\{D_t^\alpha f(t)\} = (j\omega)^\alpha \mathcal{F}\{f(t)\}, \tag{1}$$

where $j = \sqrt{-1}$, ω and $\mathcal{F}\{\cdot\}$ denote the Fourier variable and operator, respectively, and $(j\omega)^\alpha = |\omega|^\alpha \exp\left(j\text{sgn}(\omega)\frac{\alpha\pi}{2}\right)$.

In applications it is possible to adopt two alternative strategies for implanting $D^\alpha f$, one in the discrete-time domain using the $\mathcal{Z}$ transform and a series expansion, and one in the frequency domain using the $\mathcal{F}$ transform and adopting a recursive pole/zero placement.

Alain Oustaloup [28–30] proposed an electrical *analogy* with the breakwater for modeling the interface between liquid and solid materials (see Fig. 2). The water distribute their energy in the rocky fractal material of the breakwater. In this analogy, the water pressure p and flow q have for analogs the electrical voltage v and current i, while the hydraulic pipes and alveoli are modeled by means of resistances R and capacitors C (see Fig. 3). Moreover, the fractal nature of the breakwater is modeled in the electrical analogy by means of series of recursive relationships of resistances and capacitances such that branches k and $k+1$ are related by the constants $\varepsilon, \eta \in \mathbb{R}^+$ such that $R_{k+1} = \frac{1}{\varepsilon}R_k$, $C_{k+1} = \frac{1}{\eta}C_k$. Therefore, the electrical current is given by $I = \sum_{k=0}^{n} I_k$ and the input admittance becomes:

$$Y(j\omega) = \frac{I(j\omega)}{V(j\omega)} = \sum_{k=1}^{n} Y_k = \sum_{k=1}^{n} \frac{sCR\dfrac{1}{(\eta\varepsilon)^{k-1}} + 1}{sC\dfrac{1}{\eta^{k-1}}}. \tag{2}$$