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Research paper

Radial and circular synchronization clusters in extended starlike network of van der Pol oscillators

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ABSTRACT

We consider extended starlike networks where the hub node is coupled with several chains of nodes representing star rays. Assuming that nodes of the network are occupied by nonidentical self-oscillators we study various forms of their cluster synchronization. Radial cluster emerges when the nodes are synchronized along a ray, while circular cluster is formed by nodes without immediate connections but located on identical distances to the hub. By its nature the circular synchronization is a new manifestation of so called remote synchronization [33]. We report its long-range form when the synchronized nodes interact through at least three intermediate nodes. Forms of long-range remote synchronization are elements of scenario of transition to the total synchronization of the network. We observe that the far ends of rays synchronize first. Then more circular clusters appear involving closer to hub nodes. Subsequently the clusters merge and, finally, all network become synchronous. Behavior of the extended starlike networks is found to be strongly determined by the ray length, while varying the number of rays basically affects fine details of a dynamical picture. Symmetry of the star also extensively influences the dynamics. In an asymmetric star circular cluster mainly vanish in favor of radial ones, however, long-range remote synchronization survives.

1. Introduction

Networks emerging due to some sort of growth process often acquire so called scale-free structure. It occurs when the growth obeys the preferential connection rule: already highly connected nodes obtain a new connection with higher probability compared to those having a small number of links [1]. This is also known as Matthew effect or cumulative advantage, i.e., advantage tends to give rise further advantage and the rich tends to get richer [2]. This process results in power law distribution of node degrees, and the resulting structures are called scale-free networks.

Scale-free networks, preferential growth and related power laws attract much of interest of researchers for almost two decades [3–5]. Perc [6] surveys Matthew effect in empirical data ranging from patterns of scientific collaboration and growth features of socio-technical and biological networks to the evolution of the most common words and phrases. Scale-free networks formed by biological cells signaling pathways and regulatory mechanisms are surveyed by Albert [7]. Traffic control in large networks, self-similarity in traffic behavior and scale-free characteristic of related complex networks are studied by

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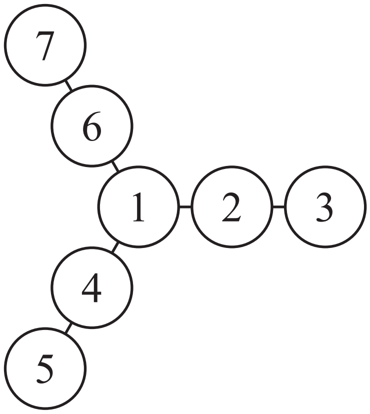

Fig. 1. Extended starlike network with $R = 3$ rays each of length $L = 2$.

Dobrescu and Ionescu [8]. Barabási et al. [9] analyze scale-free properties of world-wide web. Sohn [10] introduces scale-free networks as powerful tools in solving various research problems related to Internet of Thinks. Bianconi and Rahmede [11] consider a generalization of networks, so called simplicial complexes, and investigate the nature of the emergent geometry of complex networks in relation with preferential growth rules and scale-free distributions of connectivity degree. Bianconi [12] analyzes further challenges and perspectives of scale-free network theory.

Populating nodes of a network with oscillators we obtain a dynamical system with highly nontrivial properties. One of its key effects is synchronization [4,13–16]. Full, phase or more subtle forms of synchronization can be observed; it can involve the whole bunch of nodes or the nodes can form synchronized clusters [5,17,18]. Network synchronization phenomena are extensively studied [14,15,19,20]. Jalan et al. [21] analyze cluster synchronization in presence of a leader. Wang and Chen [22] investigate the control of a scale-free dynamical network by applying local feedback injections to a fraction of network nodes. Covariant Lyapunov vectors [23] and their nonwandering predictable localization on nodes of scale-free networks of chaotic maps are studied by Kuptsov and Kuptsova [24]. Li et al. [25] investigate the quantum synchronization phenomenon of a scale-free network constituted by coupled optomechanical systems.

The main motif of scale-free networks is a star. This structure includes a single hub node connected with several peripheral ones. The peripheral nodes form star rays. Nodes that belong to different rays are not connected with each other. Since the stars can be treated as building blocks for scale-free networks, they attracts a lot of interest in literature. Chaotic synchronization of oscillator networks with starlike couplings is considered by Pecora [26]. Ma et al. [27] study formation of synchronized clusters in such networks and derive a sufficient condition of existence and asymptotic stability of a cluster synchronization invariant manifold. Kuptsov and Kuptsova [28] show that starlike networks of chaotic maps can demonstrate wild multistability, i.e., multistability including hardly predictable number of states with narrow basins of attraction. For such networks the generalization of master stability function approach is developed and synchronized clusters of chaotic nodes are studied [29]. Chacón et al. [30] show that periodic pulses can be used to effectively control of chaos in starlike networks. Synchronization of starlike network of fractional order nonlinear oscillators is studied by Wang and Zhang [31]. Hutton and Bose [32] analyze a quantum system of spins coupled as a starlike network.

One of specific features of starlike networks is so called remote synchronization. It emerges in networks of nonidentical self-oscillators as a phase synchronization of peripheral nodes when the hub is not synchronized with them. Bergner et al. [33] study this effect for periodic oscillators on compact stars with only one node per ray. Similar effect in starlike networks of chaotic oscillators is also known and called relay synchronization [34–36]. Besides, the remote synchronization is reported for scale-free networks [37,38]. Jalan and Amritkar [39,40] observe similar regime for scale-free networks of chaotic maps and call it driven synchronization.

In this paper we consider extended starlike networks where the hub is coupled with several chains of nodes of equal lengths. Fig. 1 demonstrates an extended star with $R = 3$ rays each of $L = 2$ nodes. The total number of nodes of an extended star is $N = RL + 1$. Assuming that the nodes are occupied by nonidentical self-oscillators we study various forms of their cluster synchronization. The nodes can be synchronized along a ray. We refer to it as radial synchronization. For example in Fig. 1 the radial synchronization occurs when the node two is synchronized with node three, or it can be nodes four and five. Also a circular synchronization can be observed when synchronized clusters are formed by nodes without immediate connections but located on identical distances to the hub. In Fig. 1 the circular synchronization can occur between near-hub nodes with numbers two, four, and six, and also between far-end nodes three, five, and seven. By the nature, the circular synchronization is a new manifestation of the mentioned above remote synchronization. Unlike the case considered by Bergner et al. [33], in this paper a long-range form of the remote synchronization is reported when the synchronized nodes can communicate only through at least three node chains. The cluster of synchronized far ends of rays can appear when other nodes remain non-synchronized, or when near-hub nodes form another synchronized cluster. Moreover, far ends can