Commun Nonlinear Sci Numer Simulat 50 (2017) 193–200

Contents lists available at ScienceDirect

Commun Nonlinear Sci Numer Simulat

journal homepage: www.elsevier.com/locate/cnsns

Research paper

Fractal approach towards power-law coherency to measure cross-correlations between time series

Ladislav Kristoufek

Institute of Information Theory and Automation, Czech Academy of Sciences, Pod Vodarenskou Vezi 4, CZ-182 08, Prague 8, Czech Republic

ARTICLE INFO

Article history:
Received 22 December 2016
Accepted 21 February 2017
Available online 1 March 2017

Keywords:
Power-law coherency
Power-law cross-correlations
Correlations

ABSTRACT

We focus on power-law coherency as an alternative approach towards studying power-law cross-correlations between simultaneously recorded time series. To be able to study empirical data, we introduce three estimators of the power-law coherency parameter H_ρ based on popular techniques usually utilized for studying power-law cross-correlations – detrended cross-correlation analysis (DCCA), detrending moving-average cross-correlation analysis (DMCA) and height cross-correlation analysis (HXA). In the finite sample properties study, we focus on the bias, variance and mean squared error of the estimators. We find that the DMCA-based method is the safest choice among the three. The HXA method is reasonable for long time series with at least 10^4 observations, which can be easily attainable in some disciplines but problematic in others. The DCCA-based method does not provide favorable properties which even deteriorate with an increasing time series length. The paper opens a new venue towards studying cross-correlations between time series.

© 2017 Elsevier B.V. All rights reserved.

1. Introduction

Analyzing fractal properties of time series has been a significant contribution of physics to a broad range of other disciplines [1–15]. In time series, fractal properties translate directly into specific correlation structures. Hurst exponent H, as a characteristic parameter of fractal series, provides an insight into asymptotic scaling of the auto-correlation function, specifically its power-law decay. In the case of stationary series, we have $0 \leq H < 1$ with a separating point of $H = 0.5$ characteristic for uncorrelated series. Persistent or long-range correlated series with $H > 0.5$ follow local trends but still remain mean reverting and stationary, while anti-persistent series with $H < 0.5$ are distinctive by their excessive switching (with respect to uncorrelated processes) [16].

Recently, the methodological framework has been generalized into the bivariate setting so that not only long-range correlations but also long-range cross-correlations can be studied using methods developed in physics [17–23]. The most popular ones have been the detrended fluctuation analysis (DFA) [24,25] and the detrended cross-correlation analysis (DCCA or DXA) [26–28] as its bivariate generalization. The development of DCCA has motivated others in introducing alternative methods such as the detrending moving-average cross-correlation analysis (DMCA) [29,30] and the height cross-correlation analysis (HXA) [31]. These estimators of the bivariate Hurst exponent H_{xy} provide an additional detail about power-law scaling of the cross-correlation function between series. Interpretation of H_{xy} is usually approached through its comparison to Hurst exponents of the separate series, i.e. H_x and H_y [31–33].

E-mail addresses: kristouf@utia.cas.cz, kristoufek@ies-prague.org

http://dx.doi.org/10.1016/j.cnsns.2017.02.018
1007-5704/© 2017 Elsevier B.V. All rights reserved.

As the next step in utilizing the fractal methods, the scale-specific correlation coefficients based on DCCA and DMCA have been proposed [34–36] as well as estimators of regression parameters for specific scales [37,38]. An important topic, which has been only slightly touched by Podobnik et al. [39] but not further developed, is the scaling of scale-specific correlations. However, the link between correlations scaling and bivariate Hurst exponent provides an important insight into the dynamics between time series.

Here we focus on this link in detail, building on the notion of squared spectrum coherency and translating it into the time domain, which is more frequent in the topical literature. We propose three estimators of the power-law coherency H_ρ based on popular time domain power-law cross-correlations methods – detrended cross-correlation analysis, detrending moving-average cross-correlation analysis and height cross-correlation analysis – and analyze their finite sample properties. This approach presents a new way how to look at and analyze dependence between simultaneously recorded series with applications across various disciplines.

2. Power-law coherency

We start with a definition of the squared spectrum coherency. For two processes $\{x_t\}$ and $\{y_t\}$ with existing power spectra $f_x(\omega)$ and $f_y(\omega)$, and cross-power spectrum $f_{xy}(\omega)$ at frequency $0 < \omega \leq \pi$, the squared spectrum coherency is defined as

$$K_{xy}^2(\omega) = \frac{|f_{xy}(\omega)|^2}{f_x(\omega)f_y(\omega)}. \tag{1}$$

If the two processes are power-law correlated so that $f_x(\omega) \propto \omega^{1-2H_x}$ and $f_y(\omega) \propto \omega^{1-2H_y}$ close to the origin ($\omega \to 0+$) with respective Hurst exponents H_x and H_y, and in addition, the processes are power-law cross-correlated so that $|f_{xy}(\omega)| \propto \omega^{1-2H_{xy}}$ close to the origin with bivariate Hurst exponent H_{xy}, we have

$$K_{xy}^2(\omega) \propto \frac{\omega^{2(1-2H_{xy})}}{\omega^{1-2H_x}\omega^{1-2H_y}} = \omega^{-4(H_{xy}-\frac{H_x+H_y}{2})} \equiv \omega^{-4H_\rho}. \tag{2}$$

as frequency ω approaches zero. We define the power-law coherency through parameter H_ρ as $H_\rho = H_{xy} - \frac{H_x+H_y}{2}$ to respect previous discussions about the relationship between the bivariate Hurst exponent H_{xy} and an average of the separate Hurst exponents [32,33,40–42]. Note that the squared spectrum coherency $K_{xy}^2(\omega)$ is restricted in the same way as the squared correlation, i.e. $0 \leq K_{xy}^2(\omega) \leq 1$ for all frequencies ω [43]. This yields only two possible settings for the exponent – either $H_\rho = 0$ or $H_\rho < 0$ [33]. The former implies that the squared coherency goes to a constant and $H_{xy} = \frac{H_x+H_y}{2}$. And the latter implies that as ω approaches zero so does the squared coherency but here specifically in the power-law manner and it is thus referred to as the power-law coherency, or anti-cointegration [32], and it has $H_{xy} < \frac{H_x+H_y}{2}$. In words, such power-law coherent processes might be correlated in the short term (at high frequencies or low scales) but they are uncorrelated in the long term perspective (at low frequencies or high scales).

The squared spectrum coherency for frequency ω can be easily translated from the frequency domain to the time domain as a squared correlation for scale $s = \frac{\pi T}{\omega_j}$ parallel to frequency $\omega_j = \frac{2\pi j}{T}$ with $j = 1, 2, \ldots, \frac{T}{2}$ as

$$\rho_{xy}^2(s) = \frac{|\sigma_{xy}(s)|^2}{\sigma_x^2(s)\sigma_y^2(s)}, \tag{3}$$

where $\rho_{xy}^2(s)$ is a squared correlation between $\{x_t\}$ and $\{y_t\}$ for scale s, and $\sigma_{xy}(s)$, $\sigma_x^2(s)$ and $\sigma_y^2(s)$ represent the scale-specific covariance and variances, respectively. The notion of power-law coherency then translates perfectly with the only difference that it occurs at high scales as a parallel to low frequencies. Specifically, we have $\sigma_x^2(s) \propto s^{2H_x}$ and $\sigma_y^2(s) \propto s^{2H_y}$ for power-law correlated processes $\{x_t\}$ and $\{y_t\}$ [16,44], and if the processes are power-law cross-correlated, we additionally have $\sigma_{xy}(s) \propto s^{2H_{xy}}$ for $s \to +\infty$ [45]. Substituting into Eq. 3, we obtain

$$\rho_{xy}^2(s) \propto \frac{s^{4H_{xy}}}{s^{2H_x}s^{2H_y}} = s^{4(H_{xy}-\frac{H_x-H_y}{2})} \equiv s^{4H_\rho}, \tag{4}$$

i.e. we have the same scaling exponent for both time ($s \to +\infty$) and frequency ($\omega \to 0+$) domain power-law coherency.

3. Estimators

In this section, we recall the essentials of the bivariate Hurst exponent estimators of interest – detrended cross-correlation analysis (DCCA), detrending moving-average cross-correlation analysis (DMCA), and height cross-correlation analysis (HXA) – and we introduce procedures to estimate the power-law coherency parameter H_ρ based on these.

3.1. Bivariate Hurst exponent estimators

In the DCCA procedure [26], let us consider two time series $\{x_t\}$ and $\{y_t\}$ with $t = 1, \ldots, T$. Their respective profiles $\{X_t\}$ and $\{Y_t\}$, defined as $X_t = \sum_{i=1}^t (x_i - \bar{x})$ and $Y_t = \sum_{i=1}^t (y_i - \bar{y})$, for $t = 1, \ldots, T$, are divided into overlapping boxes of length s

Download English Version:

https://daneshyari.com/en/article/5011480

Download Persian Version:

https://daneshyari.com/article/5011480

Daneshyari.com