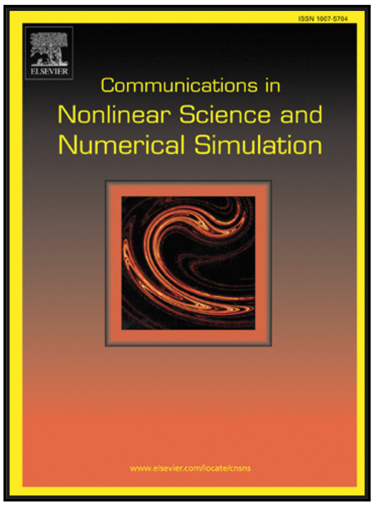

Efficient Modified Chebyshev Differentiation Matrices for Fractional Differential Equations

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PII: S1007-5704(17)30056-4
DOI: 10.1016/j.cnsns.2017.02.009
Reference: CNSNS 4116

To appear in: *Communications in Nonlinear Science and Numerical Simulation*

Received date: 9 May 2016
Revised date: 10 January 2017
Accepted date: 7 February 2017

Highlights

- An integration method is described for solving a system of linear fractional differential equations of commensurate or incommensurate order by using FCDMs.
- The proposed method is based on fractional collocation differentiation matrices (FCDMs). Moreover, three FCDMs based on finite differences are first proposed and compared with Podlubnys matrix previously used in the literature, after which two new efficient FCDMs based on Chebyshev collocation are proposed.
- The aforementioned spectral fractional operational matrices have been obtained based on direct implementation of a well-known property of fractional differentiation of polynomial bases and called *direct-FCDMs* in this paper.
- The limitations of direct-FCDMs are shown both analytically and numerically, and a method without any symbolic manipulations in multi-precision arithmetic is proposed to reduce round-off errors.
- The main contribution of the paper is proposing a novel method to obtain a new fractional derivative operational matrix (called a "fast-FCDM" here) explicitly by a modified Chebyshev differentiation matrix.
- In contrast to direct methods where the gamma function appears in each coefficient of the interpolating polynomial, the fast-FCDM does not use the gamma function. As a result, the proposed fast-FCDM is not limited to the size or the number of the collocation points.
- It is shown that the approximation of the fractional derivative via the fast-FCDM is significantly more efficient and accurate than the ones obtained by direct methods.
- The stability region of the numerical method using the proposed FCDMs are studied.
- The stability region of the numerical method using the fast-FCDMs is more accurate than the one given by the other FCDMs.
- The advantages of the proposed fast-FCDM for solving FDEs are demonstrated in seven objective examples and compared to the results obtained using the other FCDMs discussed previously. i.e.
  - In Example 7.1, fractional derivatives of a function by using the proposed FCDMs are compared.
  - In Example 7.2, the limitation of using the direct implementation method to obtain FCDMs is discussed.
  - In Example 7.3, different FCDMs are used to obtain the solution of a FDE with Caputo or RL operators, and then the results are compared.
  - In Example 7.4 illustrates the true advantages of using fast-FCDMs in solving a FDE with a highly oscillatory Mittag-Leffler solution.
  - In Example 7.5, the stability region of an incommensurate linear second order FDE in a parametric plane is shown.
  - In Example 7.6, the solution of a nonlinear FDE is obtained by using the proposed method.
  - In Example 7.7, the solution of a time-fractional diffusion equation with non-zero boundary conditions is obtained by using the proposed FCDMs.