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Research paper

Recurrence quantity analysis based on singular value decomposition

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ABSTRACT

Recurrence plot (RP) has turned into a powerful tool in many different sciences in the last three decades. To quantify the complexity and structure of RP, recurrence quantification analysis (RQA) has been developed based on the measures of recurrence density, diagonal lines, vertical lines and horizontal lines. This paper will study the RP based on singular value decomposition which is a new perspective of RP study. Principal singular value proportion (PSVP) will be proposed as one new RQA measure and bigger PSVP means higher complexity for one system. In contrast, smaller PSVP reflects a regular and stable system. Considering the advantage of this method in detecting the complexity and periodicity of systems, several simulation and real data experiments are chosen to examine the performance of this new RQA.

1. Introduction

In nature and social life, we always encounter recurrence phenomenons in diverse aspects. On small scales, we may think of the firing of neurons, on intermediate scales of climate change, on very large scales of the motions of plants or galaxies. Recurrence is a fundamental property of many dynamical systems. In case the phase space trajectory runs by a region it passed before, we call it a recurrence. Henri Poincare, introduced the formal concept of recurrence in the work from 1890s about the restricted three-body system [1]. From the 1970s to 1980s, with the intense usage of computers, different terms like dot plot, contact map, similarity matrix or distance matrix were proposed [2–7].

In 1987, the method of recurrence plot was introduced by Eckmann et al. to visualize recurrences of phase space trajectories by utilization of the similarity matrix as a tool [8]. For a dynamical system, suppose we have a trajectory $\{\overrightarrow{x_i}\}_{i=1}^{N}$ in the phase space, then the recurrence plot is obtained from the following recurrence matrix:

$$\mathbf{R}_{i,j} = \begin{cases} 1 & \overrightarrow{x_i} \approx \overrightarrow{x_j} \\ 0 & \overrightarrow{x_i} \not\approx \overrightarrow{x_j} \end{cases} \quad i, j = 1, \ldots, N \tag{1}$$

N represents the number of considered states and $\overrightarrow{x_i} \approx \overrightarrow{x_j}$ means equality with a threshold value or error ε. If a state is similar with another, the matrix R is indicated by 1. On the contrary, if the states are different, the corresponding matrix R is 0. RP has been made use of in numerous fields such as economy, engineering [9,10], biology [11,12], astrophysics [13,14], neuroscience [15–17], earth sciences [18–20] etc. Actually, apart from RP, there are many other findings based on recurrence like: recurrence time statistics [21,22], first return map [23], space time separation plot or recurrence-based measures for the

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detection of non-stationarity [24]. In addition to this, some works have been done to study complex network and topology structure [25–27].

Considering the original definition of recurrence plot, the neighborhood is a ball. However the neighborhood should depend on the purpose of the analysis. For instance the well-known cross recurrence plot (CRP) [28] is mostly used with the fixed amount of nearest neighborhood. Therefore, further modifications and extensions of RP have been introduced in the literature including: global recurrence plot [29], unthresholded recurrence plot [30], corridor thresholded recurrence plot [31], perpendicular recurrence plot [32], iso-directional recurrence plot [33], windowed and meta recurrence plot [34,35], order patterns recurrence plots [36,37].

However RP is just a tool for visualization, and it is hard to provide quantitative analysis for dynamical systems by detecting the patterns and structures based on the recurrence plot. To overcome the disadvantages, Zbilut and Webber introduced the density of recurrence point of RP in 1992 [38] and developed the recurrence quantification analysis (RQA) based on diagonal lines, vertical lines and horizontal lines in the following several years [39].

The simplest measure of the RQA is the recurrence rate (RR) which is a measure of the density of recurrence points in the recurrence plot. The next measures are based on diagonal lines like: determinism (DET), average diagonal line length (L), divergence (DIV), entropy (ENTR), trend (TREND) and ratio (RATIO). On the other hand, there are some measures based on vertical lines including laminarity (LAM), trapping time (TT) and the maximal length of the vertical lines (V_{max}). Some typical patterns in RP can be detected by these measures of the RQA, especially chaos-order transitions and chaos-chaos transitions [40].

Next we turn to another basic and important tool: singular value decomposition (SVD). In the 1870s, the SVD was established for real square matrices by Beltrami and Jordan [41], for complex square matrices by Autonne [42], and for general rectangular matrices by Eckart and Young [43]. In linear algebra, the singular value decomposition (SVD) is a factorization of a matrix and it has many useful applications in series processing and statistics. The singular value is non-negative, and for symmetric matrix, the singular value is equal to eigenvalue. There are many applications that employ the SVD including computing the pseudoinverse, least squares fitting of data, multivariable control, matrix approximation, and determining the rank, range and null space of a matrix [44–47]. For some data and signal processing, the variables are correlative and some of them can be expressed by others. People want to find the part of them which can express the whole. SVD is a typically effective method to get the core components and weight of each component.

In this work, we will start with phase space reconstruction and get the recurrence plot. Then we calculate the singular values of recurrence matrix. As it is known, the numbers of bigger singular values and their weights indicate the core components and their frequencies. Then, analogous to some popular recurrence quantification analysis measures, the distribution of singular values will be discussed and principal singular value proportion (PSVP) will be proposed. Classic recurrence quantification analysis (RQA) is developed based on the measures of recurrence density, diagonal lines, vertical lines and horizontal lines. PSVP will study the RP of original systems based on singular values.

This paper is organized as follows. In Section 2, this paper presents a brief review of some concepts and definitions including: reconstruction of phase space, recurrence plot and singular value decomposition. Then, we will study RP based on singular values and propose the principal singular value proportion (PSVP). In Section 3, several typical simulation sequences like Gaussian White Noise and Logistic map, will be chosen to evaluate the performance of the new recurrence analysis model. In Section 4, normal ECG along with CHF-ECG data, and traffic flow data of one week will be used for examining the application effects of PSVP in reality.

2. Methods

2.1. Reconstruction of phase space

In order to analyze the time series deriving from a system and view the underlying dynamics of the system, making a reconstruction of phase space is often necessary. This tool has various applications including noise reduction [48,49], signal classification [50], control [51] and nonlinear prediction [52–55].

Recall that we have a time series with the length N, $X = \{x_1, x_2, \ldots, x_N\}$. After choosing the time delay τ and embedding dimension d, the original time series is reconstructed into a matrix:

$$X_{N-(d-1)\tau,m} = \begin{bmatrix} x_1 & x_{1+\tau} & \cdots & x_{1+(d-1)\tau} \\ x_2 & x_{2+\tau} & \cdots & x_{2+(d-1)\tau} \\ \vdots & \vdots & \cdots & \vdots \\ x_{N-(d-1)\tau} & x_{N-(d-2)\tau} & \cdots & x_N \end{bmatrix} \tag{2}$$

Then the problem is how to choose the time delay τ and embedding dimension d. For time delay, mutual information (MI) [56] and autocorrelation [57] are the most widely used methods. In this paper we use mutual information to gain time delay τ for the original time series. For embedding dimension, we employ the method of false nearest neighbors (FNN) [58], which is one of the principal method of choosing embedding dimension.