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Research paper

On a predator-prey reaction-diffusion model with nonlocal effects☆

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ABSTRACT

In this paper, we consider an initial value problem of a predator-prey system with integral term. By establishing comparison principle and constructing monotone sequences, the existence and uniqueness for the solution of that problem is proved. Then we further show the uniform boundedness. Finally, some conditions of Turing bifurcation occurring are given and illustrated by numerical simulations.

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1. Introduction

This paper is devoted to the following predator-prey system

$$\begin{cases} u_t = u_{xx} + ru\Big(1 + \alpha u - \beta u^2 - (1+\alpha-\beta)\int_{\mathbb{R}} \phi(x-y)u(y)dy\Big) \\ \qquad -uv, \quad (x,t) \in \mathbb{R} \times (0,\infty), \\ v_t = v_{xx} + av(u-b), \quad (x,t) \in \mathbb{R} \times (0,\infty), \\ u(x,0) = u_0(x), \quad v(x,0) = v_0(x), \quad x \in \mathbb{R}, \end{cases} \tag{1.1}$$

where ϕ, u_0 and v_0 are continuous and satisfy

$$\begin{cases} \phi(x) \geq 0, \quad \phi \in L^1(\mathbb{R}) \quad \text{and} \quad \int_{\mathbb{R}} \phi(x)dx = 1. \\ u_0(x) \geq 0, \quad v_0(x) \geq 0 \quad \text{and } u_0, v_0 \in L^\infty(\mathbb{R}). \end{cases} \tag{1.2}$$

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The parameters r, a, b, α, β are positive and respectively represent the natural growth rate, predator rate, mortality rate of prey, efficiency for prey species to aggregation and space competition. α, β satisfy $1+\alpha-\beta>0$. Prey and predator population density are denoted by u and v. In addition, the first term αu shows the advantage to individuals in aggregating or grouping. The term $-\beta u^2$ indicates competition for space while integral term $-(1+\alpha-\beta)\int_{\mathbb{R}}\phi(x-y)u(y)dy$ can be regarded as the competition for food among individuals. avu and $-abv$ respectively denote predation benefit and death of predator in the absence of prey. For more information about predator-prey model, we can refer to [18,21,24,25].

The pioneering work of problem (1.1) could be originated from Gourley and Britton [8,9]. They considered the following system

$$\begin{cases} u_t = D\triangle u + u[1+\alpha u-(1+\alpha)(G**u)] - uv, \\ v_t = \triangle v + av(u-b) \end{cases} \tag{1.3}$$

analyzed the linear stability of the coexistence states of (1.3) and obtained that bifurcations occurred from it. Here, $(x,t)\in\mathbb{R}^n\times(0,\infty)$ and $(G**u)(x,t)=\int_{\mathbb{R}^n}\int_{-\infty}^{t}G(x-y,t-s)u(y,s)dsdy$. It is noted that the equation about the prey in system (1.3) can't well refine the competition of intra-species. Thus, Gourley [11] explored the following equation about the prey

$$u_t = du_{xx} + ru\left(1+\alpha u-\beta u^2-(1+\alpha-\beta)\int_{\mathbb{R}}\phi(x-y)u(y)dy\right), \quad (x,t)\in\mathbb{R}\times(0,\infty). \tag{1.4}$$

For a special kernel $\phi(x)=\frac{1}{2\lambda}e^{-\frac{|x|}{\lambda}}$, they studied the traveling wave solutions of Eq. (1.4) in the case of weak nonlocal (λ is sufficiently small). Further, Billingham [3] considered (1.4) when the nonlocality is strong (i.e. λ sufficiently large). For the general kernel, by constructing suitable super and sub-solutions for a two point boundary value problem and combining Schauder fixed point, Han and Wang [16] proved that (1.4) exists traveling wave solutions connecting 0 to an unknown positive steady state. In addition, Gourley [10], Brestycki et.al [2], Hamel et.al [15], Fang et.al[13], and Alfaro et.al [1] included the results about traveling waves and steady states of (1.4) as $\alpha=\beta=0$.

Recently, there are some great progresses on the solution of (1.4) with initial value $u(x,0)=u_0(x)$. By establishing the comparison principle, constructing super- and sub-solutions, Deng [5] showed that the solution of this initial problem is existed, unique and persist. Moreover, Deng [6,7] considered the general case

$$\begin{cases} u_t = du_{xx} + u[f(u)-\alpha(g*u)], \quad (x,t)\in\mathbb{R}\times(0,+\infty), \\ u(x,0)=u_0(x), \quad x\in\mathbb{R}, \end{cases}$$

where f is a continuous differentiable function on $(0,\infty)$ and satisfies some conditions.

Up to now, investigations about problem (1.1) are still seldom. Xu et al. [20] recently studied the pattern dynamics of (1.1) with spatiotemporal delay. By using the Turing instability analysis, they gave the linear stability conditions of Turing stability and simulated the Turing patterns. Whereas, since the comparison principle does not hold for the problem (1.1), there have no any results about the properties of the solution of (1.1) (note that the challenge is different if the integral term occurs in different position in the system, that is why the research about (1.1) seldom occurred). Inspired by Deng and Wu [5–7], we here consider the properties of the solution about the initial problem (1.1), including the existence, uniqueness, boundedness and Turing bifurcation. In order to do that, we will give a new definition of super and sub-solution for (1.1) and use some auxiliary functions. With such definition, a comparison principle can be established. And then we can construct monotone sequences of super and sub solutions, which will lead to the existence of the solution through passing to the limit. Making use of an auxiliary function, the uniform boundedness of the corresponding solution of system (1.1) can be transformed into that of a single equation.

This paper is organized as follows. In Section 2, we provide some preparatory works. The existence and uniqueness of the solution are proved in Section 3. Finally in Section 4, we show some properties of the solution, including the boundedness and the Turing bifurcation.

2. Comparison principle

In this section, we will give some preparatory works. First, we introduce the definition of super- and sub-solutions of system (1.1). Then, the comparison principle is established.

For convenience, let $I_T=\mathbb{R}\times(0,T)$ and $I_T\cup B_T=\mathbb{R}\times[0,T)$. Now, we give the following definitions of super- and sub-solution of system (1.1).

Definition 2.1. If $(\overline{u}(x,t),\overline{v}(x,t))$ and $(\underline{u}(x,t),\underline{v}(x,t))$ satisfy

(i) $\overline{u}$, $\overline{v}$, $\underline{u}$, $\underline{v}\in C^{2,1}(I_T)\cap C_B(I_T\cup B_T)$ and $\overline{u}(.,t)$, $\overline{v}(.,t)$, $\underline{u}(.,t)$, $\underline{v}(.,t)\in L^1(-\infty,\infty)$, where C_B is a bounded continuous space;

(ii) $\overline{u}(x,0)\geq u_0(x)\geq\underline{u}(x,0)$ and $\overline{v}(x,0)\geq v_0(x)\geq\underline{v}(x,0)$ for $x\in\mathbb{R}$;

(iii) for any $(x,t)\in I_T$,

$$\begin{cases} \overline{u}_t \geq \overline{u}_{xx} + r\overline{u}\left(1+\alpha\overline{u}-\beta\overline{u}^2-(1+\alpha-\beta)\int_{\mathbb{R}}\phi(x-y)\underline{u}(y)dy\right) - \overline{u}\underline{v}, \\ \overline{v}_t \geq \overline{v}_{xx} + a\overline{v}(\overline{u}-b), \end{cases} \tag{2.1}$$

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