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Research paper

Break-even concentration and periodic behavior of a stochastic chemostat model with seasonal fluctuation

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A R T I C L E I N F O

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A B S T R A C T

This paper formulates a single-species stochastic chemostat model with periodic coefficients due to seasonal fluctuation. When the noise is small, a modified break-even concentration is identified, whose value below or above the averaged concentration of the input nutrient can completely determine whether the microorganism will persist or not, where an accuracy decay rate is given for extinction. In case of persistence, existence of the random positive periodic solution is proved for the considered model. Further, the random periodic solution is shown to be globally attractive under some mild extra condition. The periodic dynamics obtained in this paper are supported by computer simulations.

1. Introduction

The well-known and mathematically simple model used to describe the continuous culture of microorganisms in a chemostat can be described by the ordinary differential equation:

$$\begin{cases} dS = \left[D\left(S^0 - S\right) - \mu(S)x\right]dt, \\ dx = [\mu(S) - D]xdt, \end{cases} \tag{1}$$

where S and x stand for the concentrations of the nutrient and the microorganism at time t respectively, see e.g. [1,2]. S^0 and D are positive constants, representing respectively the input concentration of the nutrient and the common washout rate. The growth rate function $\mu(S)$ is non-negative, which have two classical forms including the uninhibited growth rate $\mu(S) = \frac{mS}{a+S}$ (see [3]), and the inhibited growth rate $\mu(S) = \frac{mS}{a+S+bS^2}$ (see [4]). To make the model be more realistic, the culture vessel charged with two or more types of microorganisms has been suggested, and known mathematically [5–7] and experimentally [8] that only one survives, that is, the competitive exclusion principle holds. In [9,10], it is interesting to find that coexistence of the predators occurs in the form of a limit cycle under appropriate circumstances by using a bifurcation theorem. A new class of observers for chemostat model using hidden symmetries is designed by Didi, Dib and Cherki [11], which has nice properties such as its general form of the gain and adjustability of the rate of convergence. Dimitrova and Krastanov [12] established sufficient conditions for global asymptotic stability to one of the equilibrium points. extending the applicability of the concept for biological control of the chemostat model. In these literatures, an underlying assumption for the studied models are that the stochastic effects can be neglected or averaged out due to the law of large numbers, and thus they neglect the random effects.

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In the real world, random effects may be neglected for large population sizes and under homogeneity conditions. But, at all other scales or when the homogeneity conditions are not met, random effects cannot be neglected, see [13]. Stephanopoulos et al. [14] considered a two populations chemostat with random fluctuations of the dilution rate, like $D \to D + \sigma \dot{B}(t)$ that allows coexistence, where $B(t)$ is a standard Brownian motion on the complete probability space $\left(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t \geq 0}, P\right)$ with the intensity $\sigma^2 > 0$. Grasman and De Gee [15] proposed a stochastic chemostat model by using the singular perturbation methods to the corresponding Fokker-Planck equation. Imhof and Walcher [16] derived the stochastic model by adding a centered Gaussian term to the deterministic system, and also proved that random effects may lead to extinction in scenarios where the deterministic model predicts persistence. In [13], Campillo et. al. build a model that still relies on a mass balance principle and that encompasses the useful stochastic information from the microscopic scale to the macroscopic scale. Zhang and his coauthors [17,18] considered the continuous flow bioreactor model, and studied the long time behavior of the corresponding stochastic models. Fritsch, Harmand and Campillo [19] presented a modeling approach to build bridges between discrete/random and continuous/deterministic under assumption the population size is large, which could be used to prove the convergence in distribution of the former models toward the latter ones, and to propose new method for modeling and simulation. Originated from a parameter estimation problem, in [20], Campillo and his coauthors found that the density can be given by the solution of the complete Fokker-Planck equation, and further designed the finite differences numerical scheme under a probabilistic viewpoint. From the view of the parameter randomization, Zhang and Jiang [21] formulated a chemostat model with Holling type II functional response supposing that the nutrient S and the microorganism x are perturbed by different noises, and found out sufficient conditions for the principle of competitive exclusion. A chemostat is a common laboratory apparatus used to culture microorganisms, the input of the growth medium and the washout rate can be artificially controlled at a given rate, the growth rate $\mu(S) = \frac{mS}{a+S}$ and m can not be obtained directly. From the experimental point of view, Ulrich Sommer [22] has given a way to estimate the parameter μ and m by using the regression method with 95% confidence intervals, which reveals that the parameter m exhibits random fluctuations to a greater or lesser extent. Therefore, it is necessary to study the influences of the noise when the maximal growth rate m is effected by the environmental noise. From this point, Xu and Yuan [23] considered the effects of the random noises on the maximal growth rate m such that $m \to m + \sigma \dot{B}(t)$, to give

$$\begin{cases} dS = \left[D(\Lambda - S) - \dfrac{mSx}{a+S}\right]dt - \dfrac{\sigma Sx}{a+S}dB(t), \\ dx = \left[\dfrac{mS}{a+S} - D\right]xdt + \dfrac{\sigma Sx}{a+S}dB(t). \end{cases} \tag{2}$$

The authors showed that (2) has a unique positive periodic solution, and then discussed the stochastic permanence and extinction under suitable conditions. Subsequent to [23], the authors [24] presented one global threshold by using the Feller test, which can be used to easily determine the persistence and extinction of the microorganism, and further showed existence of the stationary distribution.

However, in reality, the periodic phenomena is yet an important factor in modeling the population growth. In order to better simulate the ecological situation in natural lakes, Hsu [25] introduced a periodically varying resource to account for changing pattern in the watershed as the seasons vary, instead of a constant input of limiting nutrient. He also represented an interesting and potentially complex situation of the two-species one-resource system, in which case, both species can handily survive. Wang and Pang [26] considered the two populations model competing for a periodically pulsed inputting nutrient with Beddington-DeAngelis growth rates, and the criteria are derived for the coexistence or non-coexistence of the competing species. The condition for stability of periodic solutions has been given in [27] to a distributed parameters biochemical system with periodic input and deterministic small perturbations. For more about the periodic effect on the dynamics of the chemostat, we refer to [26–30] and cited therein. Very recently, Wang, Jiang and O′Regan [31] proposed a stochastic chemostat model with periodic washout rate where the nutrient and the microorganism are influenced by different environmental noises, and then showed the conditions for the existence of the nontrivial positive periodic solution and the boundary periodic solution.

Motivated by the above discussions, based on model (2), we aim to study the single-species stochastic chemostat model with periodic coefficients of the form

$$\begin{cases} dS(t) = \left[D(t)\left(S^0(t) - S(t)\right) - \dfrac{m(t)S(t)x(t)}{a(t)+S(t)}\right]dt - \dfrac{\sigma(t)S(t)x(t)}{a(t)+S(t)}dB(t), \\ dx(t) = \left[\dfrac{m(t)S(t)}{a(t)+S(t)} - D(t)\right]x(t)dt + \dfrac{\sigma(t)S(t)x(t)}{a(t)+S(t)}dB(t). \end{cases} \tag{3}$$

The main concerns of this paper are as follows:

- Under what conditions will microorganism survive or be washed out?
- When does the random periodic and positive solution exist?
- To examine whether random periodic solution is attractive.

The organizations of the paper are as follows. In Section 2, we prove some basic results of (3) and also introduce the important lemmas. The results on extinction and persistence are separately established in Section 3 by choosing appropriate