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Research paper

Size-dependent pull-in voltage and nonlinear dynamics of electrically actuated microcantilever-based MEMS: A full nonlinear analysis

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ABSTRACT

This paper aims to develop a size-dependent nonlinear model for electrically actuated microcantilever-based MEMS based on the modified couple stress theory. The pull-in instability and nonlinear dynamics of the microcantilever are explored considering the full nonlinear equation of motion. The discretized equations are obtained using Galerkin method. It has been demonstrated that the material's small length scale parameter and geometric nonlinearities significantly influence the static and dynamic pull-in behaviors of the microcantilever-based MEMS. In the presence of the length-scale parameter, the pull-in voltage is found to be size-dependent. If a time-dependent harmonic component is superposed on the DC voltage, the primary resonances of the micro-cantilever are observed. When the dimensionless length-scale parameter is relatively large, the frequency-response curves indicate that the dynamic responses of the microbeam can evolve from softening-type to hardening-type nonlinear behaviors; the combined effects of length-scale parameter and geometric nonlinearities on the pull-in band of frequencies may be remarkable.

1. Introduction

It has been reported that microelectromechanical systems (MEMS) have important applications in the fields of biomedicine, aerospace, information technology, etc. [1]. Among various applications, electrically actuated microbeams have become one of the core structures in MEMS since the electrically actuated microbeams have low power consumption, simple configuration and good technique compatibility [2]. It is not surprising, therefore, that the topic on the behavior of electrically actuated microbeams has attracted much attention during the past two decades.

There are a number of actuation methods for MEMS devices [3]. Perhaps electrostatic actuation is the most well established among various actuation methods due to its simplicity and high efficiency [4]. By using this method, the microbeam is deflected by a DC (Direct Current) voltage and can be driven to oscillate around its natural frequency by an AC (Alternate Current) harmonic load [3]. During this process, the microbeam may be subjected to a static instability (also known as static pull-in instability) when the applied voltage exceeds a critical value (dubbed pull-in voltage). If, however, a harmonically perturbed voltage is present, the so-called dynamic pull-in instability can occur under certain conditions [3]. The pull-in phenomenon forms the basis of operation for radio-frequency (RF) MEMS switches [5], in which the structure is

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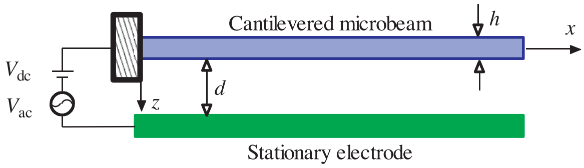

Fig. 1. Schematic of a cantilevered microbeam under an electric actuation.

actuated by a voltage load beyond the pull-in voltage to snap the structure with fast speed and minimal time. Therefore, a better understanding of the pull-in instability mechanism and dynamical behaviors of electrically actuated microbeams is helpful for developing safety standard in the design of MEMS.

A large number of studies (see, e.g., [6–9]) have addressed the pull-in phenomenon and dynamical behaviors of electrically actuated microbeams/microplates. The knowledge gained enables designers to better avoid/use such static or dynamic pull-in instability phenomena. The literature on the static and dynamic responses of electrically actuated microbeams may be grouped into two: the classical beam models (see, e.g., [3,9]) and the size-dependent beam models (see, e.g., [1, 10–14]).

For the pull-in analysis of electrically actuated microbeams, many earlier studies were based on the classical beam theories while the small scale effects on the performance of microbeams were not accounted for. However, the classical beam models based on the conventional continuum mechanics could not sufficiently predict the static and dynamic responses of microbeams in the case where size effects are essentially non-negligible. Indeed, the size-dependent mechanical behaviors have been frequently observed in experiments [15–17]. Recently, therefore, several non-classical elasticity theories have been used for better characterizing the behaviors of electrically actuated microbeams. The modified couple stress theory and strain gradient elasticity theory have become a possible choice for developing size-dependent theoretical models for microbeams [1,13,18,19], while the nonlocal elasticity theory and surface elasticity theory were reported to be more suitable for nanobeams [20].

In the past years, both cantilevered and supported microbeams with electrical forces have been reevaluated using size-dependent beam models. The main findings based on non-classical beam models indicate that the static and dynamic responses of electrically actuated microbeams are size-dependent and hence the pull-in voltages are generally higher than those predicted by classical beam models.

For microbeams with both ends positively supported, it was reported that both mid-plane stretching and electric load cause nonlinear terms that can profoundly affect the responses of the electrically actuated microbeams [21]. Yin et al. [1] analyzed the static pull-in instability of an electrostatically actuated microbeam with both end supported using a modified couple stress theory; their study showed that the static deflection is size-dependent. Ghayesh and his co-workers [10] have further studied the nonlinear primary resonances of supported microbeams under electrical force. They found that the supported system displays a weaker softening nonlinear behavior and a smaller amplitude for the case when the modified couple stress theory is employed.

For cantilevered microbeams under electrical force, although there were a few papers (see, e.g., [1,13,22,23]) dealing with the size-dependent pull-in mechanism of the system, most of these work deals with the statics by neglecting the geometric and/or inertial nonlinearities. When the microbeam's deflection becomes relatively large, the effects of the geometric and inertial nonlinearities on the dynamic pull-in behaviors of the cantilever have not been revealed. This motivates the current work.

This paper is planned as follows. In Section 2, based on the modified couple stress theory, a new, full nonlinear equation of motion of the electrically actuated microbeam with clamped-free boundary conditions is derived using the Hamilton's principle. Unlike other theoretical models for cantilevered microbeams, the resulting analytical model contains not only the material length scale parameter, but also the geometric and inertial nonlinearities. To illustrate the newly developed model, the static pull-in instability and nonlinear dynamics are analyzed in Sections 3 and 4, respectively. The paper concludes with a summary in Section 5.

2. A full nonlinear equation of motion for cantilevered systems

The purpose of this section is to derive a full nonlinear equation of motion for the cantilevered microbeam under electrical actuation, when the motion is large. This derivation is closely following, but not identical to that of Semler [24].

A schematic diagram for the physical system is shown in Fig. 1. It is assumed that the microbeam is elastic. The rotatory inertia and shear deformation are neglected in the current study. Since the microbeam is clamped-free, the centerline is taken to be inextensible. The micro-cantilever has length L, width b, thickness h, and classical flexural stiffness EI. This deformable microbeam is separated by a dielectric space with an initial gap d from the stationary electrode. For application purpose, the microbeam is undergoing an electric load $V_{dc} + V_{ac}\cos\Omega t$, where V_{dc} is the static voltage and V_{ac} is the perturbed amplitude of the AC voltage. In addition, Ω is the excitation frequency of the AC voltage.

As plotted in Fig. 2, in the case of lateral vibrations of the cantilevered microbeam, two coordinate systems may be introduced: the Eulerian (x, z) and the Lagrangian (x_0, z_0). Consider the undisturbed axis of the microbeam to be horizontal,