Commun Nonlinear Sci Numer Simulat 46 (2017) 126–134

Contents lists available at ScienceDirect

Commun Nonlinear Sci Numer Simulat

journal homepage: www.elsevier.com/locate/cnsns

Research paper

Emergent dynamics of spiking neurons with fluctuating threshold

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ARTICLE INFO

Article history:
Received 3 July 2015
Revised 25 October 2016
Accepted 29 October 2016
Available online 4 November 2016

Keywords:
Fluctuating threshold
Normal probability distribution
Coupled neurons
Synchronization

ABSTRACT

Role of fluctuating threshold on neuronal dynamics is investigated. The threshold function is assumed to follow a normal probability distribution. Standard deviation of inter-spike interval of the response is computed as an indicator of irregularity in spike emission. It has been observed that, the irregularity in spiking is more if the threshold variation is more. A significant change in modal characteristics of Inter Spike Intervals (ISI) is seen to occur as a function of fluctuation parameter. Investigation is further carried out for coupled system of neurons. Cooperative dynamics of coupled neurons are discussed in view of synchronization. Total and partial synchronization regimes are depicted with the help of contour plots of synchrony measure under various conditions. Results of this investigation may provide a basis for exploring the complexities of neural communication and brain functioning.

1. Introduction

Spike initiation is a manifestation of excitable neurons which occurs as a result of depolarization of membrane potential above a critical value (known as threshold). Spiking neurons exhibit a variety of firing patterns characterizing different neuronal phenomena [1,2]. Variation in spiking may be due to the effect of the nature of the stimulus or the activation, inactivation process of various voltage gated ion channels [3]. The underlying mechanism of spike emission and its propagation is well explained by Hodgkin and Huxley in their experiment on squid giant axon [4]. Investigation of spatiotemporal variation of spikes reveals nonlinear characteristics of slow-fast neuronal dynamics. Essentially, neuronal communication is mediated by the spikes along nerve cells. There are various models of excitable neurons which capture major firing behavior of real neurons [4–6]. The threshold value for triggering spike is assumed to be constant in each of these aforesaid models [7–9]. However, in real neuron, the threshold is found to be dynamic and dependent on previous behavior [10] although the exact reason of the origin of threshold variability is not clearly known [3]. The nonlinear nature of temporarily varying threshold has been shown to be important for information processing in neurons [11–14]. Neuronal firings are reported to be spiking at regular or random intervals due to the inherent mechanism or due to the interaction with the neighbouring neurons or external stimulus [15–17]. This leads to change in the modal characteristics of the spike intervals. In the present work we have investigated the response of an excitable neuron associated to a normally distributed time varying threshold. The probability distributions of Inter Spike Intervals (ISI) and the threshold are compared under various conditions. The

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http://dx.doi.org/10.1016/j.cnsns.2016.10.022
1007-5704/

probability distribution of ISIs is found to be unimodal for smaller fluctuation in the spiking threshold which on the other hand, seen to be bimodal with further increase in fluctuation. The investigation is further extended from single neuron to coupled neurons with normally distributed thresholds, as intricate tasks are performed through a network of interactive neurons in brain with cognition and reliability. The collective dynamics of these interactive neurons is supposed to play vital role in information processing and transfer. Studies on synchronization claim that alpha and gamma EEG rhythms are the effect of partial synchrony in the cortical neurons [18]. However, the synchronization of spiking neurons is reported to induce epileptic seizure, Parkinson disease etc [19].

The synchronization has been described as a characteristic of an interactive network when the dynamics of one neuron is followed by the others. Mathematically, a network of N coupled neurons is said to be fully synchronized if the difference between any two trajectories converges to zero after the transients are over i.e.$\lim_{t\to\infty} |X_i(t) - X_j(t)| = 0$ for all i, $j = 1,2,\ldots,N$, $i \neq j$ where $X_i(t)$ is the state variable of a neuron.

In our investigation, we have considered a system of two coupled FHN neurons interacting with each other via diffusive coupling. The cases of instantaneous and delayed coupling are discussed in view of synchronization which is measured as [20,21]:

$$\chi^2(N) = \frac{\sigma_X^2}{\frac{1}{N}\sum_{i=1}^{N}\sigma_{X_i}^2} \tag{1}$$

where, $\sigma_X^2 = [X(t)]^2 - [X(t)]^2$ measures the fluctuation of collective membrane potential, $X(t) = \frac{1}{N}\sum_{i=1}^{N} X_i(t)$ and $\sigma_{X_i}^2 = [X_i(t)]^2 - [X_i(t)]^2$, $i = 1,2,\ldots N$. Computed $\chi^2(N)$ results in 0 for asynchrony and 1 in case of full synchrony; it lies within [0,1] in all other cases.

Synchronization in the present case is explained by the phase plot of the membrane potentials of the coupled neurons. Moreover, $\chi^2(N)$ is computed for different choices of the parameters e.g., coupling strength (D), threshold variation (S) and the time delay (τ). Number of neurons N is taken as 2 in this case.

2. Mathematical models

(A) Single neuron model

We have considered the following FHN model of single neuron involving membrane potential (v) and recovery variable (w):

$$\frac{dv}{dt} = a[-v(v-1)(v-b) - w + I] \tag{2a}$$

$$\frac{dw}{dt} = v - cw \tag{2b}$$

Here, I refers to the externally applied current, a and c are scaling parameters and b is the fluctuating threshold which is normally distributed. With a suitable choice of the parameters, the time course of membrane potential of the neuron described by the above set of equations (Eq. (2)) exhibits continuous spikes. In our case, the mean value of the firing threshold is fixed at b_0 and it is then allowed to fluctuate randomly with a control parameter S. The system of Eq. (2) is solved numerically where in each step the threshold variable b is defined as follows:

$$b(t) = b_0 + S\,R(t) \tag{2c}$$

where $R(t)$ is the randomness induced in the system with zero mean and unit standard deviation.

(B) Coupled neuron models

Cognitive activities in the brain are performed due to the collective dynamics of an ensemble of interactive neurons. The interaction takes place via coupling of two or more neurons. The interaction in the network often renders the neurons to synchronize and synchronization has been found to play a crucial role in information processing and transfer among different brain areas for physiological functions [22,23]. The present study considers two diffusively coupled FHN neurons as follows:

$$\frac{dv_1}{dt} = a[-v_1(v_1-1)(v_1-b_1) - w_1 + I + D(v_2(t-\tau) - v_1)] \tag{3a}$$

$$\frac{dw_1}{dt} = v_1 - cw_1 \tag{3b}$$

$$\frac{dv_2}{dt} = a[-v_2(v_2-1)(v_2-b_2) - w_2 + I + D(v_1(t-\tau) - v_2)] \tag{3c}$$