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Research paper

Dynamics of traveling waves in fluctuating nonlocal media

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ABSTRACT

The article deals with nonlocal hydrodynamic models for structured media with a fluctuating parameter. We are interested in the structure of traveling wave solutions disturbed by noise. Using the stochastic sensitivity function technique, the confidence ellipses for periodic trajectories obeying the period doubling scenario, hidden and spiral periodic orbits are derived. To identify the peculiarities of confidence ellipses, we consider the variation of eccentricity and area over the period of a periodic trajectory. We show that the dynamics of eccentricity of noisy limit cycle, up to triple period, has the number of minima coinciding with the cycle's multiplicity, whereas this is not in the case of quadruple cycle. The profiles of function for the areas of confidence ellipses characterize the heterogeneous anatomy of stochastic attractors and possess scaling properties for multiple cycles. Considering the eccentricity and area of confidence ellipses for the spiral trajectory existing in the vicinity of Shilnikov homoclinic loop, the intensive oscillations of eccentricity and area are observed when the confidence ellipses are derived for the flow near the one dimensional manifold of Shilnikov's orbit.

1. Introduction

The appearance of new properties and unusual behavior in natural and artificial materials under high intensive loading causes the increasing amounts of physically significant inhomogeneities and changing the character of interaction between them [1]. This is valid for granular media, soils, colloidal solutions, biologic tissues, populations of organisms and etc. In such media when they are far from equilibrium the new sources of nonlinearity appear, the processes of relaxation are exited, the auxiliary degrees of freedom emerge, and the interactions between subsystems become nonlocal. Describing these peculiarities of intrinsic construction of medium in a mathematical model, we arrive to a notion "structured medium".

Within the framework of hydrodynamic description, which can be applied to the structured medium in the long wave approximation, we can leave the continuity equation and the equation of motion in their classical form and all information about the medium's structure incorporate into the equation of state. To construct the proper equation of state, the concept of strong nonequilibrium open thermodynamic systems is fruitful. In particular, to take the processes of relaxations and nonlocal interaction between structural elements into account, we use the additional internal variables [5], provided with the proper equations of kinetics. Using the ideas of the irreversible thermodynamics, we can reduce the model for the structured medium to the hydrodynamic type system describing the relaxing media with spatially nonlocal effects and

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having the following form [2,5,16]

$$\frac{d\rho}{dt}+\rho\frac{\partial u}{\partial x}=0, \qquad \rho\frac{du}{dt}+\frac{\partial p}{\partial x}=\gamma\rho,$$
$$\tau\left(\frac{dp}{dt}-\chi\frac{d\rho}{dt}\right)=\kappa\rho-p+\sigma\left[\frac{\partial^2 p}{\partial x^2}+\frac{1}{\rho}\frac{\partial p}{\partial x}\frac{\partial \rho}{\partial x}-\chi\left(\frac{\partial^2\rho}{\partial x^2}-\frac{1}{\rho}\left(\frac{\partial\rho}{\partial x}\right)^2\right)\right], \tag{1}$$

where ρ and p are the density and pressure, u is the velocity, $\gamma\rho$ is the mass force. The parameter τ denotes the time of relaxation. The parameters κ and χ are proportional to the square of equilibrium and frozen velocities of sound in a medium, $\frac{d(\cdot)}{dt}=\frac{\partial(\cdot)}{\partial t}+u\frac{\partial(\cdot)}{\partial x}$.

Unlike the classical hydrodynamic model when $\sigma=\tau=0$, model (1) possesses the wide range of traveling wave solutions describing the pattern formation in structured media [2,11].

In addition to the aforementioned description of structured medium, real systems involve fluctuations, explorations of which become often the primary purpose of the studies. In particular, seismic noise contains the information on the geology of Earths crust, its temporal and spatial dynamics [21,22]. The analysis of this noise provides the development of the seismic emission tomography [22] and the surface wave tomography [23]. The change of spectrum of microseismic noise can be regarded as an earthquake precursor [24] partially due to the phenomena of prebifurcation noise amplification and noise-dependent hysteresis [25]. Noise can also produce non-equilibrium phase transition to an ordered symmetry-breaking state [26], enhances the wave propagation in sub-excitable media [27], the spatiotemporal phenomena, including traveling waves, in discrete media [32], and the modifications of classical scenario of bifurcations or even the creation of new their types [7,33].

Thus, for the deep understanding of self-organizing phenomena, pure deterministic description needs to be accompanied by the incorporation of effects having the probabilistic nature [6].

We are going to take into account external fluctuations in model (1), introducing the parameters perturbed by noise. This approach allows us to reduce the model to a low dimensional stochastic dynamical system with additive or multiplicative noise and use a large number of methods applied to deterministic models [4].

This report is organized as follows. In Section 1 the wave solutions for deterministic model (1) and results of their qualitative analysis are presented. In Section 3 we introduce the stochastic model and describe the tools for numerical treatments. Our findings concerning the studies of periodic orbits with the help of shooting method and stochastic sensitivity function technique are presented in Section 4. The discussion of results obtained and concluding remarks are contained in the final section.

2. Qualitative analysis of the set of traveling wave solutions to the deterministic model

Let us recall the main results of qualitative analysis of periodic traveling wave solutions to deterministic model (1). Using the group theory methods, it easy to check that model (1) admits the operator [3,5]

$$\hat{R}=\frac{\partial}{\partial t}+D\frac{\partial}{\partial x}+\xi\left(\rho\frac{\partial}{\partial\rho}+p\frac{\partial}{\partial p}\right),$$

where the parameters D and ξ are constants.

The invariants of operator $\hat{R}$ allow us to construct the wave solutions to model (1) in the following form

$$u=U(s)+D,\ \rho=\rho_0\exp\left(\xi t+S(s)\right),\ p=\rho Z(s),\ s=x-Dt, \tag{2}$$

where now the parameter D can be identified with the velocity of wave front, the meaning of ξ [3,10] is discussed below. Substitution (2) transforms system (1) to the quadrature $\frac{dS}{ds}=-\frac{W+\xi}{U}$, $W=\frac{dU}{ds}$ and the dynamical system

$$U\frac{dU}{ds}=UW, \quad U\frac{dZ}{ds}=\gamma U+\xi Z+W\left(Z-U^2\right),$$
$$U\frac{dW}{ds}=\left\{U^2\left[\tau\left(\gamma U+\xi Z-WU^2\right)+\chi\tau W+Z-\kappa\right]\right.$$
$$\left.+\,\sigma\left[(\xi+W)(2U(\gamma-UW)+\chi W)+U^2W^2\right]\right\}\left[\sigma\left(\chi-U^2\right)\right]^{-1}. \tag{3}$$

This system has only one nontrivial fixed point with the coordinates

$$U_0=-D, \quad Z_0=\frac{\kappa D^2}{D^2-2\sigma\xi^2}, \quad W_0=0 \tag{4}$$

under the auxiliary condition $\gamma=\frac{\xi Z_0}{D}$. Fixed point (4) corresponds to the steady solution $u=0,\ p=Z_0\rho,\ \rho=\rho_0\exp(\xi x D^{-1})$. The parameter ξ thus characterizes the level of inhomogeneity of steady solution.

Using the Andronov–Hopf theorem [20], the conditions of periodic orbits creation were derived. In particular, for the fixed parameters $\chi=50, \tau=0.1, \xi=1.8, \sigma=0.76$ and $\kappa=18.8$, using the Dormand-Prince method [17], the development of limit cycle is displayed in Fig. 1a at decreasing D^2 where the periodic doubling cascade leading to chaotic attractor