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Research paper

Mixed-mode oscillations in a nonlinear time delay oscillator with time varying parameters

Yue Yu[a], Xiujing Han[b], Chun Zhang[c], Qinsheng Bi[b,*]

[a] School of Science, Nantong University, Nantong, 226007, PR China
[b] Faculty of Civil Engineering and Mechanics, Jiangsu University, Zhenjiang, 212013, PR China
[c] School of Mathematical Science, Huaiyin Normal University, Huaiyin, 223300, PR China

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ABSTRACT

In this study, the mechanism for the action of time-invariant delay on a non-autonomous system with slow parametric excitation is investigated. The complex mix-mode oscillations (MMOs) are presented when the parametric excitation item slowly passes through critical bifurcation values of this nonlinear time delay oscillator. We use bifurcation theory to clarify certain generation mechanism related to three complex spiking formations, i.e., “symmetric sup-pitchfork bifurcation”, “symmetric sup-pitchfork/sup-Hopf bifurcation”, and “symmetric sup-pitchfork/sup-Hopf/homoclinic orbit bifurcation”. Such bifurcation behaviors result in various hysteresis loops between the spiking attractor and the quasi-stationary process, which are responsible for the generation of MMOs. We further identify that the occurrence and evolution of such complex MMOs depend on the magnitude of the delay. Specifically, with the increase of time delay, the two limit cycles bifurcated from Hopf bifurcations may merge into an enlarged cycle, which is caused by a saddle homoclinic orbit bifurcation. We can conclude that time delay plays a vital role in the generation of MMOs. Our findings enrich the routes to spiking process and deepen the understanding of MMOs in time delay systems.

1. Introduction

As one of the commonest examples in nonlinear texts and research articles [1–3], the typical Duffing oscillator with free vibration is described by

$$\ddot{x}(t) + 2\mu\dot{x}(t) + (\alpha + \rho)x(t) + \beta x^3(t) = 0 \tag{1}$$

where $x(t)$ is the displacement, μ is the damping ratio, α, ρ mean linear stiffness parameters and β means cubic (nonlinear) stiffness parameter. This model exhibits many qualitatively different phenomena and is considered as one of the most intensely studied system in nonlinear system, which has served in physics, electronics, biology, neurology and so on.

In a series of paper [4,5], dynamical systems with time delays have been studied in neuronal works originally and have been a subject of intensive research. It is worth noting that the dynamical characteristics including stable, unstable, oscillatory or chaotic behavior of neural networks with time delays have drawn much attention of researchers. In Ref. [6–11],

* Corresponding author.
E-mail address: biqinshengpr@sina.cn (Q. Bi).

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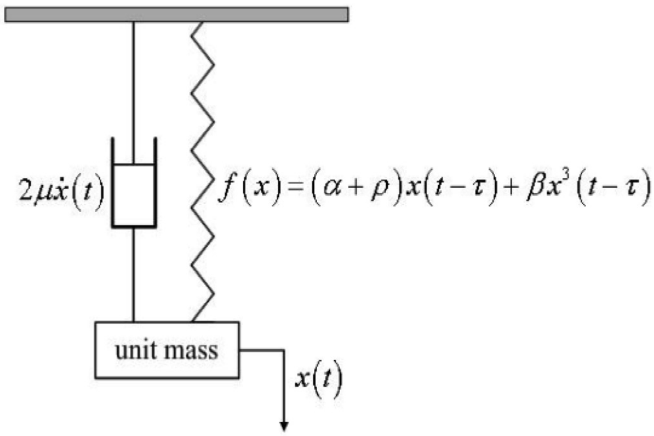

Fig. 1. A mass-spring vibration oscillator with delayed spring force related to Eq. (2).

Yutaka, Jeevarathinam, Wang and Cao et al. introduced a time-invariant delay into Eq. (1), and the following delay differential equation (**DDE**) of Eq. (2) can be obtained

$$\ddot{x}(t) + 2\mu\dot{x}(t) + (\alpha + \rho)x(t - \tau) + \beta x^3(t - \tau) = 0 \tag{2}$$

where $\tau > 0$ means a time delay.

This one-degree-of-freedom system of Eq. (2) can be represented by a real vibration system (see in Fig. 1), which has a unit mass, a viscous damping, and a restoring force expressed by the function of $f(x)=(\alpha+\rho)x(t-\tau)+\beta x^3(t-\tau)$, where $x(t)$ is displacement and t is the time. Bifurcation properties in Eq. (2) without the parametric excitation have been investigated in details [12–14], in which the Hopf bifurcation may lead to the periodic oscillations according to the appropriate time delay, and dynamical analysis of periodic solution can be discussed by the center manifold approaches [15].

Noting that most practical implementations of feedback have inherent delays, including not only delayed state feedback but also delayed velocity feedback. In our paper, we consider the following Eq. (3) in delayed controller with state feedback and velocity feedback

$$\begin{cases} \dot{x}(t) = y(t - \tau) \\ \dot{y}(t) = -2\mu y(t - \tau) - (\alpha + \rho)x(t - \tau) - \beta x^3(t - \tau) \end{cases} \tag{3}$$

For our study, we consider a typical parametrically excited self-exciting dynamic model derived from Eq. (3), i.e., $\rho = A\cos(\Omega t)$, A and Ω represent the amplitude and the frequency of the parametric excitation respectively. Many results relating to the parametric frequency with $\Omega = O(1)$, have been well studied [16,17]. However, when order gap exists between the periodic parametric excitation Ω and the natural frequency, i.e., $\Omega = O(\varepsilon)$ and $0 < \varepsilon \ll 1$, the effect of multiple time scales appears, resulting in some interesting dynamical behaviors, such as mixed-mode oscillations (MMOs) [18–21]. The oscillatory behavior of MMOs often behaves in periodic states characterized by a combination of relatively large amplitude (spiking process) and nearly harmonic small amplitude oscillations (quasi-stationary process), conventionally denoted by N^K with N and K corresponding to large and small amplitude oscillations, respectively.

Up to now, most of reports about MMOs are focused on systems without delays [22–24], while MMOs in delay system with time varying parameters have not been studied sufficiently. The MMO generating mechanism needs further study with the variation of delays [25]. Here, we take the delay as a variable parameter to investigate its effect on mixed-mode dynamic in DDE of Eq. (3) with slow varying parametric excitation item. Some novel spiking formations related to time delay are introduced, the mechanism of which is analyzed. Specially, we present the Bogdanov–Takens (**BT**) bifurcation induced by the delay, which leads to the MMOs generation near homoclinic orbits. We reveal that MMOs' occurrence and evolution in delayed system critically depend on the delay magnitude itself as well as its induced bifurcations.

The organization of the rest of the paper is as follows. In Section 2, characteristics of the parametrically excited model with a time-invariant delay are discussed. An analysis of bifurcations and dynamics in fast subsystem is obtained as a function of the parametric excitation term. Among them, the existence of Bogdanv–Takens bifurcation and the saddle homoclinic orbits are verified, which allows us to make precise predictions about the occurrence of mixed-mode dynamic in next section. In Section 3, under slowly parametric excitation, various types of MMOs with complicated waveforms are presented in this model, where time delay can be considered as a tuning parameter to generate or regulate MMOs. Finally, in Section 4 we briefly summarize the results and offer the conclusion words.