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Research paper

Delay-induced oscillations in a thermal convection loop under negative feedback control with noise

Dmitri Bratsun[a,*], Ivan Krasnyakov[a], Alexey Zyuzgin[b]

[a] Department of Applied Physics, Perm National Research Polytechnical University, Perm, 614990, Russia
[b] Department of General Physics, Perm State National Research University, Perm, 614990, Russia

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ABSTRACT

We study both experimentally and theoretically the problem of active control of the mechanical equilibrium of a fluid in a convection loop heated from below and cooled from above. In order to easily obtain and maintain the mechanical equilibrium of fluid we have designed a rectangular-shaped loop with long vertical channels and short crosspieces between them. The control is performed by using a negative feedback subsystem which inhibits the convection by introducing small discrete changes in the spatial orientation of the loop with respect to gravity. In this paper, we focus on effects that arise when the feedback controller operates with time delays and/or is subjected to random fluctuations. Both these intrinsic features of the controller could be tuned in experiments to explore their effects together and separately. When the noise is absent, the excess feedback was found to lead to the excitation of delay-related oscillations. In addition, we show that time delay coupled with noise can cause a system to be oscillatory even when its deterministic counterpart exhibits no oscillations. So, we give an example of a hydrodynamic system having, generally, a large number degrees of freedom, which behaves like a small-sized stochastic system heavily dependent on fluctuations, even far from the point of bifurcation. The experimental data and theory is shown to be in good agreement.

1. Introduction

Dynamical systems with delay are abundant in nature and attract more and more attention of researchers. They occur in a wide variety of physical, chemical, engineering, economic and biological systems. A range of applications includes, but not limited to, population dynamics [1,2], gene regulation [3,4,36], biomedical modeling of tumor growth [5], radiophysics systems [6], control in mechanical engineering [7,8], fluid mechanics of inhomogeneous media under vibrations [9–11] etc. Time delay itself may arise due to quite different reasons, for example, the finite speed of the propagation (electrical or light signal), the presence of inertia in some system elements (in the feedback control theory), compound multistage reactions involving the sequential assembly of long molecules (in the gene regulation).

Mathematical analysis of delay differential equations (DDEs) have been considered in a large number of works, and the summary of their results can be found in monographs by Polyanin and Nazaikinskii [12] dealing with the exact solutions of DDEs, by Yi et al. [13] devoted to the application of the Lambert function for DDEs, by Hu and Wang [7] and Briat

* Corresponding author.
E-mail addresses: dmitribratsun@rambler.ru (D. Bratsun), krasnyakov_ivan@mail.ru (I. Krasnyakov), Alexey.Zyuzgin@psu.ru (A. Zyuzgin).

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[14] discussing the stability and control issues, by Lakshmanan and Senthilkumar [6] considering the general classification of DDEs and their linear and nonlinear properties.

This paper is devoted to the stabilization of the no-motion state of a fluid in a thermal convection loop via a negative feedback control affected by time-delays and noise. The convection loop sometimes referred to as a thermosyphon, a closed tube filled with fluid and externally heated, is often used in the experimental and theoretical analysis of the a convective stability. Since the paper by Welander [15], the natural circulation in a static loop has been investigated by several authors, with the main motivation being the fundamental study of simple systems exhibiting typical nonlinear convective effects. One can mention the work by Malkus [16] who proposed a model based on a set of three ODEs of the Lorenz type. Ehrhard and Müller [17] have performed a nonlinear analysis of these equations as well as experiments to validate the model predictions.

The choice of a convective loop shape is determined, on the one hand, by a relatively simple one-dimensional flow structure. On the other hand, these fluid flows are quite easy to control. The flow control issue generally excites serious interest both from fundamental perspective and in connection with many technological applications (see, for example, the papers by Zyuzgin et al. [18,19] and Bratsun et al. [20,21] discussing the topic of the external control in fluid mechanics systems). The development of technologies demands the creation of systems with automatic control, which do not require constant monitoring by a person. The main idea of active convection control is to change the state of the convective system by means of suppression or, on the contrary, strengthening naturally occurring small disturbances of flow in real time. In this case the control parameter becomes a function of time and the state of a system under the control. Thus, active feedback allows to make the control more adaptive and effective.

The research group led by Bau contributed a lot into the study of feedback control of convection [22–27]. In [22,23] they have studied a toroidal-shaped convective loop both theoretically and experimentally. The control was applied by means of side heating with the negative feedback. It is interesting to note that the setup has provided effective control over developed convection regimes, but not the mechanical equilibrium of fluid: the stabilization effect was obtained numerically, but it did not find experimental support. That is why their next paper Tang and Bau [24] analyzed the possibility of no-motion state stabilization already in a porous medium to embody the experiment for more inert and slow system. Further, they have demonstrated how one can control the Marangoni convection [25], micro-fluidic systems [26] and the Rayleigh-Bénard convection [27]. In the latter paper the authors have used the proportional and derivative controllers to postpone the transition from the no-motion state to the convective state in a circular cylinder heated from below and cooled from above. They have chosen the control strategy based on the heating from a system of individually controlled actuators whose power is adjusted in proportion to temperatures measured in the cylinder's interior. The most intriguing result is the discovery of time-wise oscillatory convection for relatively large values of the control parameter. The authors have explained that this instability was induced by large linear controller gains.

In contrast to many works treating similar topics (see, for example, [17,23,28]) we have used from very beginning a rectangular-shaped convective loop [9]. It appears that despite the attractiveness of a toroidal shape, the practical implementation there of a mechanical equilibrium state even without external control is proved to be a challenging task. Indeed, one can obtain the vertical temperature gradient by heating the loop in a very special way. Probably, this is why still no successful experimental stabilization of no-motion state in a toroidal loop reported in the literature. In contrast, we have demonstrated in [9] that by using a rectangular loop one can easily obtain and maintain for a long time the mechanical equilibrium of fluid. We have developed also a new way to control the system by means of its rotation with respect to the direction of gravity. In this case, we found that the excessive strengthening of the feedback excites periodic oscillations in the system. It was shown in [9] that oscillations are caused by the natural time delay in the controller. Thus, it turned out that we deal here with a typical system with negative time delay feedback.

As it is known, one used to distinguish between chaos and stochasticity. Stochastic processes are about ordered behaviors emerging from random systems, while term "chaos" originally referred to complex, random-seeming behaviors emerging from deterministic systems. It was found that the coupling of noise to nonlinear deterministic behavior can lead to nontrivial effects. For example, noise can excite internal modes of oscillation and can even enhance the response of a nonlinear system to external signals. This noise-induced phenomenon is called stochastic resonance [29,30]. The main idea is that a nonlinear system is subjected to a periodic modulated signal so weak as to be normally undetectable, but it becomes detectable due to resonance between the weak deterministic signal and stochastic noise.

Another important research area is the study of the effect of stochastic fluctuations on deterministic systems with a time delay. One of the first time delay model with noise was suggested by Ohira with colleagues [31–33]. This is a one-dimensional delayed random walk model which is a simple, but still retaining important properties [31]. The Ehrenfest model of a discrete random walk evolving on a quadratic potential was generalized to the case of non-zero delay in [32]. The corresponding Fokker-Planck equation was derived in [33]. They found that the characteristic feature of delayed random walk is the oscillatory behavior of its correlation function that reflects the oscillatory path by the walker. It is noteworthy that joint action of noise and delay can result in some kind of resonant phenomena as it was shown theoretically within a simple two-state model in which transition rates between states are time-delayed [34]. The combined effect of three phenomena – noise, delay and external periodic modulation was studied experimentally in [35].

At the moment, the topic of coupling of noise and time delay in various dynamic systems is extremely relevant in connection with important applications in the gene regulation and bioengineering. As it is known, the small number of reactant molecules involved in gene regulation can lead to significant fluctuations in protein concentrations. In addition, transcription processes are not just stochastic but they are rather compound multi-stage reactions involving sequential assembly of