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Research paper

Nonlinear damped oscillators on Riemannian manifolds: Numerical simulation

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ABSTRACT

Nonlinear oscillators are ubiquitous in sciences, being able to model the behavior of complex nonlinear phenomena, as well as in engineering, being able to generate repeating (i.e., periodic) or non-repeating (i.e., chaotic) reference signals. The state of the classical oscillators known from the literature evolves in the space $\mathbb{R}^n$, typically with $n = 1$ (e.g., the famous van der Pol vacuum-tube model), $n = 2$ (e.g., the FitzHugh–Nagumo model of spiking neurons) or $n = 3$ (e.g., the Lorenz simplified model of turbulence). The aim of the current paper is to present a general scheme for the numerical differential-geometry-based integration of a general second-order, nonlinear oscillator model on Riemannian manifolds and to present several instances of such model on manifolds of interest in sciences and engineering, such as the Stiefel manifold and the space of symmetric, positive-definite matrices.

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1. Introduction

Nonlinear oscillators (both autonomous and driven) have been widely studied in the scientific literature either because they arise naturally in the process of modeling complex physical structures and because they constitute the basis for several modern applications. Paradigmatic examples of nonlinear oscillators obtained from models of complex physical systems are the van der Pol oscillator, that arose from a model of vacuum tubes [29], and the Lorenz oscillator [22], that was derived from the simplified model of convection rolls in the atmosphere and has important implications in climate and weather predictions. The literature is rich in Lorenz-like systems, such as the Chen system and the Lü system [21]. Paradigmatic applications of designed oscillators (either self-sustained or controlled) is to the secure transmission of information [40], to the active damping of mechanical vibrations [9], and to the analysis of bivariate data by a coupled-oscillators approach [34]. A detailed list of applications of non-linear, chaotic, oscillators in science and engineering may be found, e.g., in the review paper [6]. We would like to cite two, in particular, that appear as especially relevant, as they relate chaos analysis with a powerful signal-processing technique known as Independent Component Analysis (ICA), namely, wearable mental-health monitoring [33], and seismic signal detection and characterization [1].

The state of nonlinear oscillators evolves over time in complex, non-repeating, deterministic patterns. Most nonlinear oscillators appear as first-order or second-order dynamical systems involving a single real variable. The simplest model is perhaps the *linear harmonic oscillator*. As no damping is present, the harmonic oscillator preserves its initial energy

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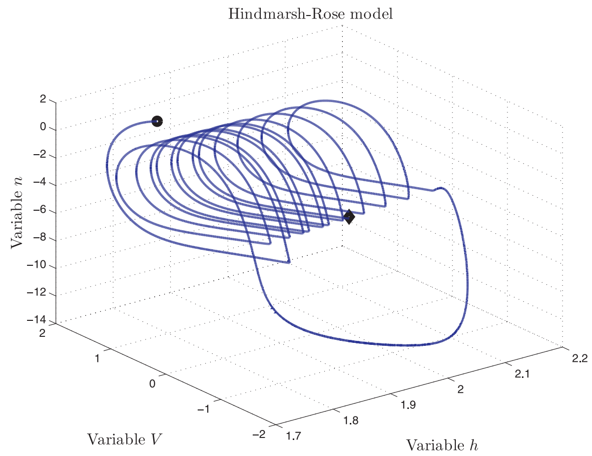

Fig. 1. Exemplary behavior of an Hindmarsh–Rose model in terms of the three variables $(V, n, h) \in \mathbb{R}^3$. The thick circle denotes the initial point, while the thick diamond denotes the final point of the trajectory.

indefinitely. An example of 'damped' oscillator is the van der Pol oscillator model, which is closely-related to biologically-inspired nonlinear dynamical systems such as the FitzHugh–Nagumo model [16], the Hodgkin–Huxley model of the activation and deactivation dynamics of spiking neurons and the Hindmarsh–Rose model [37], that augments with a slow variable the planar FitzHugh–Nagumo model. Another well-studied nonlinear system that exhibits a complex behavior is the *Duffing oscillator* that models, for example, a spring pendulum whose spring's stiffness does not exactly obey Hooke's law [36].

Examples of dynamical systems involving more than one variable are known in the scientific literature. A nonlinear, three-dimensional, deterministic dynamical system is the *Rabinovich–Fabrikant oscillator* [30]. It is described by a set of three coupled ordinary differential equations comprising two parameters, which may exhibit a complex behavior for certain values of the parameters, while for other values of the parameters its flow may tend to a stable periodic orbit. Likewise, the *Rössler oscillator* [35] helps describing equilibrium in chemical reactions. In addition, a three-dimensional nonlinear oscillator is the Colpitts circuit, built up of a bipolar junction transistor and a resonant network consisting of an inductor and two capacitors [24].

The state of the above nonlinear oscillators recalled from the scientific literature evolves in the real line $\mathbb{R}$ or in the real plane $\mathbb{R}^2$ or in the ordinary space $\mathbb{R}^3$. As an example, the Fig. 1 illustrates the state of an Hindmarsh–Rose model in terms of the three variables $(V, n, h) \in \mathbb{R}^3$. The present paper aims at extending previous studies on nonlinear autonomous oscillators from flat Euclidean spaces to high-dimensional curved Riemannian manifolds. Riemannian manifolds of interest in the literature are the Stiefel manifold (along with the special cases of the unit-hypersphere and the orthogonal group), the space of symmetric, positive-definite matrices and the special orthogonal group. In particular, the current contribution aims at presenting *discrete-time* nonlinear autonomous oscillators equations that may be implemented on a computing platform and at investigating to what extent the obtained discrete-time dynamical system replicates its theoretical continuous-time differential-geometric properties.

The theory and practice of non-linear oscillators is one of the topics of prime interest in the nonlinear science community, as testified by a number of papers about non-linear oscillators in mobile robotics [5], thermodynamics [27], signal transmission and processing [39,43], mathematical optimization [46] and artificial intelligence [44]. The motivation and fundamental aim of the present contribution is to open new perspectives in the theory of nonlinear damped oscillators on curved spaces and to promote research efforts in this field.

The current paper is organized as follows. The Section 2 recalls the notation used in differential geometry (in Subsection 2.1) and describes a general second-order dynamical system derived by the analysis of a point-wise particle sliding on a smooth manifold following the landscape of a potential energy function and under the effect of passive/active damping (in Subsection 2.2); the Subsection 2.3 illustrates the notation and the general structure of the second-order oscillator via a 1-dimensional example. The Subsection 3.1 describes discrete-time second-order autonomous oscillators on the unit-hypersphere, the Subsection 3.2 describes oscillators on a Lie group, namely, the manifold of special orthogonal matrices, the Subsection 3.3 deals with the manifold of symmetric, positive-definite matrices, and the Subsection 3.4 illustrates discrete-time second-order autonomous oscillators on the compact Stiefel manifold, for which several quantities of interest are not available in closed form; the Subsection 3.5 discusses the problem of the (lack of) conservation of energy in theoretically-conservative systems due to finite-length stepping in discrete-time systems. The Section 4 illustrates the developed theory by means of two examples of oscillators on the sphere $\mathbb{S}^2$ that allows graphical rendering. The Section 5 concludes the paper and outlines some foreseen applications and theoretical research.

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