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Research paper

Vibrational resonance in a harmonically trapped potential system

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ABSTRACT

We report our investigation of role of shape of a harmonically trapped potential system driven by a biharmonic external force with two widely different frequencies ω and Ω with $\Omega \gg \omega$ on vibrational resonance. The potential is capable of generating odd number of potential wells depending upon the values of the parameters in the potential function. Applying a theoretical approach we obtain an analytical expression for the response amplitude Q at the low-frequency ω. The response amplitude exhibits multiple peaks and approaches a non-zero limiting value when the amplitude of the high-frequency force is varied. We explain the mechanism of observed resonance dynamics. We bringout the effect of number of potential wells on the resonance behaviour.

1. Introduction

Nonlinear systems are ubiquitous and certain types of them are capable of showing a variety of phenomena. Among the various phenomena exhibited by nonlinear systems nonlinear resonances are fundamental importance in basic and applied sciences. The forced resonance (resonance induced by an additive external periodic force) [1–4] and parametric resonance [4,5] occur in both linear and nonlinear systems. The stochastic resonance [6,7], coherence resonance [8,9], auto resonance [10,11], ghost resonance [12–14], chaos resonance [15] and vibrational resonance [4,16–20] can be realized only in nonlinear systems. The present paper is concerned with vibrational resonance. Consider a nonlinear system driven by two periodic signals of widely different frequencies, say ω and Ω with $\Omega \gg \omega$. Assume that in the absence of high-frequency (Ω) force, the response amplitude of the system at the low-frequency ω is weak. When the amplitude g of the high-frequency force is varied from a small value, the response amplitude at the low-frequency ω displays resonance at one or more values of g. This high-frequency force induced resonance at the low-frequency is termed as vibrational resonance.

Gitterman [17] and Blekhman and Landa [18] developed theoretical methods for analyzing vibrational resonance. Since then this resonance phenomenon has been investigated theoretically, numerically and experimentally in many oscillators, excitable systems, maps, networks and time-delayed systems. It is important to investigate the role of shape of potentials on vibrational resonance. In this direction, the occurrence of vibrational resonance has been examined in bistable [16–18,21–24], monostable [25], asymmetric [26], spatially periodic [27] and spatially extended single-well [28] potentials systems. It has also been analyzed in an excitable system [29] and in a system with signum nonlinearity [30].

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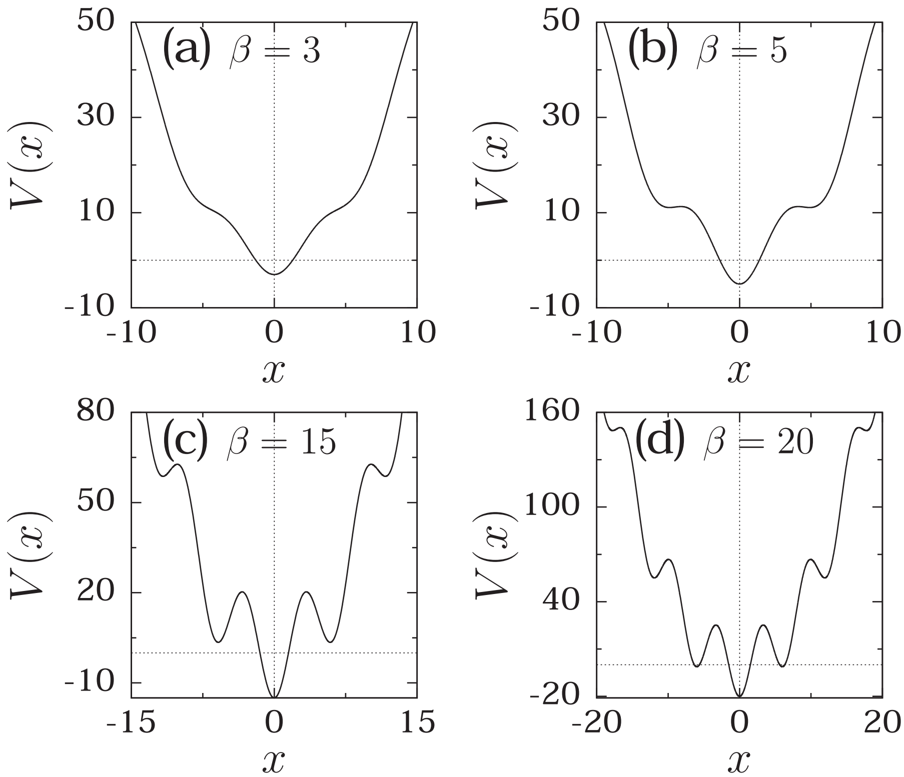

Fig. 1. The shape of the potential (1) with $\omega_0^2 = 1$ and for four values of β.

Motivated by the above consideration, in the present paper we consider the damped nonlinear system with harmonically trapped potential

$$V = \frac{1}{2}\omega_0^2 x^2 - \beta \cos x, \quad \omega_0^2,\ \beta > 0 \tag{1}$$

and driven by the biharmonic force. The equation of motion of the system of our interest is given by

$$\ddot{x} + d\dot{x} + \omega_0^2 x + \beta \sin x = f \cos \omega t + g \cos \Omega t, \quad \Omega \gg \omega. \tag{2}$$

In Eq. (2) $\Omega \gg \omega$ and $d > 0$. A feature of the potential given by Eq. (1) is that it can generate odd number of potential wells of different depths depending upon the values of the parameters ω_0^2 and β. Fig. 1 shows the shape of the potential for four fixed values of β for $\omega_0^2 = 1$. The multistable potential generated experimentally in an experimental torsion balance oscillator [31] resembles that of the multistable potential given by Eq. (1) for larger values of β. It can be used as an alternative potential for the typical polynomial potentials $V(x)$ used for representing potentials with single-well, triple-well and other odd-number of wells.

Here, we are interested in investigating the role of shape of the potential given by Eq. (1) on vibrational resonance. Applying a theoretical treatment, we obtain an approximate analytical expression for the response amplitude (Q) at the low-frequency ω. The theoretical prediction is found to be in good agreement with the numerically computed Q. Even for the single-well case of the system the response amplitude shows number of resonance peaks approaching a limiting value. We describe the mechanism of observed variation of Q.

The organization of the paper is as follows. To start with in Section 2 we present the effect of the parameter β on the shape of the potential and the number of equilibrium points. The number of equilibrium points depends on β. In Section 3 first we assume that the solution of the system consists of two frequencies ω and Ω and obtain an approximate linear equation of motion for the component of solution with period $2\pi/\omega$ or the frequency ω. We find an analytical expression for $Q(\omega)$. We compare the theoretical Q with the numerically computed Q. Then we analyse the occurrence of vibrational resonance for the cases of the system with single-well. Vibrational resonance of the system with three and five wells is discussed in Section 4. In Section 5 we present the effect of initial conditions on $Q(\omega)$ for $\beta = 15$. Finally, Section 6 contains conclusion.