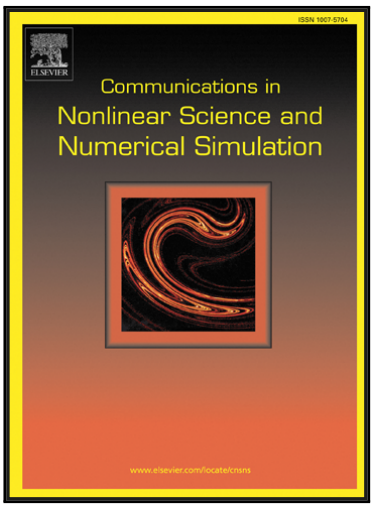

Bifurcations of phase portraits of pendulum with vibrating suspension point

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Bifurcations of phase portraits of pendulum with vibrating suspension point

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Abstract

We consider a simple pendulum whose suspension point undergoes fast vibrations in the plane of motion of the pendulum. The averaged over the fast vibrations system is a Hamiltonian system with one degree of freedom depending on two parameters. We give a complete description of bifurcations of phase portraits of this averaged system.

1 Introduction

A simple pendulum with vibrating suspension point is a classical problem of perturbation theory. The phenomenon of stabilisation of the upper vertical position of the pendulum by fast vertical vibrations of the suspension point was discovered by A. Stephenson [1, 2]. In these papers the linearisation and reduction to the Mathieu equation is used. The case of inclined vibrations of the suspension point is considered as well. Nonlinear theory was developed by N.N.Bogolyubov [3], who used the averaging method, and by P.L.Kapitsa, who has developed a method of separation of slow and fast motions for this [4, 5] (see also [6]). Different aspects of this problem were discussed in many publications (see, e.g., [7] for a discussion of geometric aspects, [8] for the case of arbitrary frequencies of vibrations, [9] for the case of random vibrations). Generalisations to double- and multiple-link pendulums are contained in [11, 12, 13]. It is noted in [10] that the problem is simplified by using averaging in Hamiltonian form. Such an approach is used, e.g., in [14, 15]. In [14] bifurcations of the phase portraits of the averaged problem for a vertically vibrating suspension point are described.

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