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Research paper

Persistence and ergodicity of plant disease model with markov conversion and impulsive toxicant input[☆]

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A B S T R A C T

Taking into account of both white and colored noises, a stochastic mathematical model with impulsive toxicant input is formulated. Based on this model, we investigate dynamics, such as the persistence and ergodicity, of plant infectious disease model with Markov conversion in a polluted environment. The thresholds of extinction and persistence in mean are obtained. By using Lyapunov functions, we prove that the system is ergodic and has a stationary distribution under certain sufficient conditions. Finally, numerical simulations are employed to illustrate our theoretical analysis.

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1. Introduction

Severe plant infectious diseases may bring about the reduction of output or total crop failure, resulting in significant losses in agricultural production. For example, the outbreak of Cocoa swollen shoot virus (CSSV) caused significant economic losses in cocoa-producing counties in West Africa, and more than 2,000,000 infectious trees had to be cut down in Ghana alone to eradicate the disease [1]. Therefore, the prevention and control of plant infectious diseases is of vital importance in agricultural production. To better understand these control strategies, numerous mathematical models have been developed in recent years [2–14]. A classical model describing plant infectious disease is in form of

$$\begin{cases} \dot{S}(t) = S(t)[r_1 - \beta I(t) - d_1 S(t)], \\ \dot{I}(t) = I(t)[-r_2 + \beta S(t) - d_2 I(t)], \end{cases} \tag{1}$$

where $S(t)$ and $I(t)$ represent the number of susceptible and infected plants at time t, respectively. All parameters are positive due to the physical meaning, and more precisely, r_1 denotes the birth rate, r_2 the death rate, β transmission rate and d_i represent density-dependence coefficients. However, model (1) can not reflect the fact that plants in nature are inevitably disturbed by various stochastic factors. Therefore, it is desired to develop new models, now known as stochastic differential

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equations(SDE) [15], to describe the effect of the external disturbance. Generally, there are two ways to achieve this, one is to use white noise and another is colored noise. Mathematically, the former can be modelled as a Brownian motion [16–19] and the latter by a finite-state Markov chain [20–30].

As suggested by references [16–18], we first in the birth and death rates take a small perturbation of the white noise into account, which results in the following

$$\begin{cases} dS(t) = S(t)[r_1 - \beta I(t) - d_1 S(t)]dt + \sigma_1 S(t)dB_1(t), \\ dI(t) = I(t)[-r_2 + \beta S(t) - d_2 I(t)]dt + \sigma_2 I(t)dB_2(t), \end{cases}$$

where $\sigma_1 > 0$ and $\sigma_2 > 0$ represent for the intensities of the white noise on the susceptible and infected respectively, and $B_i(t), i = 1, 2$ represents one-dimensional standard Brownian motion. Furthermore, taking into account the influence of colored noise to coefficients $r_i, d_i, \sigma_i, i = 1, 2$ and β yields

$$\begin{cases} dS(t) = S(t)[r_1(r(t)) - \beta(r(t))I(t) - d_1(r(t))S(t)]dt + \sigma_1(r(t)S(t)dB_1(t), \\ dI(t) = I(t)[-r_2(r(t)) + \beta(r(t))S(t) - d_2(r(t))I(t)]dt + \sigma_2(r(t))I(t)dB_2(t), \end{cases} \tag{2}$$

in which $r(t)$ is defined as a right-continuous Markov chain on the probability space $\left(\Omega, \mathcal{F}, \{\mathcal{F}_{t\geq 0}\}, \mathbb{P}\right)$, whose state space is $\mathbb{S} = \{1, 2, \cdots, m\}$.

We next consider the effect of environmental pollution since it has been widely accepted that pollutants may pose a serious risk to agricultural production, for example some of the toxicant can kill crops directly. In general, there are two ways for pollutant emission, continuously or impulsively. Now, in this paper, we consider the case that toxicant is discharged impulsively. To this end, suppose that $c_e(t)$ stands for the concentration of the toxicant in the environment at time t, τ is the impulsive input period and μ is the impulsive input amount. Then, system (2) becomes the following stochastic differential system with impulsive toxicant input

$$\begin{cases} \left.\begin{aligned} dS(t) = S(t)[r_1(r(t)) - \beta(r(t))I(t) - d_1(r(t))S(t) \\ \quad -\beta_1(r(t))c_e(t)]dt + \sigma_1(r(t))S(t)dB_1(t), \\ dI(t) = I(t)[-r_2(r(t)) + \beta(r(t))S(t) - d_2(r(t))I(t) \\ \quad -\beta_2(r(t))c_e(t)]dt + \sigma_2(r(t))I(t)dB_2(t), \\ \frac{dc_e(t)}{dt} = -hc_e(t), \end{aligned}\right\} t \neq n\tau, \\ \triangle S(t) = 0, \triangle I(t) = 0, \triangle c_e(t) = \mu, t = n\tau, \end{cases} \tag{3}$$

where h denotes the loss rate of toxicant at time t, β_1 and β_2 represent the death rate of susceptible and infected plants caused by the toxicant, respectively. For any $k \in \mathbb{S}$, $r_i(k)$, $d_i(k)$, $\beta_i(k)$, $\sigma_i(k)$ $(i = 1, 2)$ and $\beta(k)$ are all positive constants.

Mathematical ecological models considering polluted environment have been active in recent years [31,32]. However, literatures taking both impulsive toxicant input and random perturbation into account are still rare. To study the dynamic behaviors of the stochastic impulsive differential system (3), the rest of the paper is organised as follows. Preliminaries are provided in Section 2. In Section 3, we explore the conditions for extinction and permanence of system (3) and provide the thresholds for extinction and permanence in mean. In Section 4, the ergodicity of system (3) is studied based on Lyapunov functions. Lastly, we use numerical simulation to illustrate our theoretical results.

2. Preliminaries

In this paper, we suppose that $(\Omega, \mathcal{F}, \mathcal{F}_{t\geq 0}, \mathbb{P})$ is a complete probability space with a filtration $\mathcal{F}_{t\geq 0}$, and $B_i(t)$ $(i = 1, 2)$ is an one-dimensional Brownian motion on this space and $r(t)$ is a right-continuous Markov chain and independent of the Brownian motion $B_i(t)$. The state space of this Markov chain is $\mathbb{S} = \{1, 2, \cdots, m\}$ as before. Further, suppose that the generator matrix of $r(t)$ is $\Gamma = (\gamma_{ij})_{1\leq i,j\leq m}$, where γ_{ij} stands for the transition rate from state i to j and satisfies the following conditions,

$$P\{r(t + \Delta t) = j | r(t) = i\} = \begin{cases} \gamma_{ij}\Delta t + o(\Delta(t)), & i \neq j, \\ 1 + \gamma_{ij}\Delta t + o(\Delta(t)), & i = j, \end{cases}$$

where $\gamma_{ij} > 0$ if $i \neq j$, while $\gamma_{ii} = -\sum_{j\neq i}\gamma_{ij}$, for $i, j = 1, 2, \cdots, m$. Since system (3) can switch between any two environmental regimes, we can suppose that Markov chain $r(t)$ is irreducible. That is, for any $i, j \in \mathbb{S}$, there are finite numbers $i_1, i_2, \cdots, i_k \in \mathbb{S}$ such that $\gamma_{i,i_1}\gamma_{i_1,i_2}\cdots\gamma_{i_k,j} > 0$. Under this condition, $r(t)$ has a unique stationary distribution $\pi = (\pi_1, \pi_2, \cdots, \pi_m) \in R^{1\times m}$, which is the solution of linear equation $\pi\Gamma = 0$, $\Sigma_{j=1}^m \pi_j = 1$ and $\pi_j > 0$ for all $j \in \mathbb{S}$. Here, Markov chain $r(t)$ is also called ergodicity. Hence, for any vector $\psi = (\psi(1), \cdots, \psi(m))^T$,

$$\lim_{t\to\infty} \frac{1}{t}\int_0^t \psi(r(s))ds = \sum_{k\in\mathbb{S}} \pi_k\psi(k).$$

Consider the SDE with Markov conversion as follows [15,20]

$$\begin{cases} dx(t) = f(x(t), r(t))dt + g(x(t), r(t))dB(t), \\ x(0) = x_0, r(0) = r_0, \end{cases} \tag{4}$$

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