



## Research paper

# Conservation laws, modulation instability and rogue waves for the localized magnetization with spin torque



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## ARTICLE INFO

## Article history:

Received 26 July 2016  
 Revised 28 October 2016  
 Accepted 2 January 2017  
 Available online 3 January 2017

## PACS:

05.45.Yv  
 04.30.Nk  
 42.81.Dp

## Keywords:

Localized magnetization  
 Spin torque  
 Rogue waves  
 Conservation laws  
 Modulation instability  
 Spatial-temporal structures

## ABSTRACT

Under investigation in this paper is an integrable equation which can describe the localized magnetization with spin torque under the long-wavelength approximation. In order to obtain the rogue wave solutions, a new Lax pair is derived. Infinitely-many conservation laws are also constructed. Based on the generalized Darboux transformation, the first-, second- and third-order rogue wave solutions are derived. Property of the rogue waves are analyzed and influence of parameters  $\alpha$ ,  $\beta$  and separating function  $f(\epsilon)$  on the rogue waves and spatial-temporal structures are also discussed (the meaning of  $\alpha$  and  $\beta$  can be found in the paper). For the case of  $f(\epsilon) = 0$ , the modulus of the  $k$ th-order rogue wave ( $k = 1, 2, 3$ ) is irrelevant to parameter  $\alpha$ . Parameter  $\beta$  influences the spatial-temporal range where the rogue wave appears. Spatial-temporal range enlarges with the increase of  $\beta$ . In addition,  $\beta$  also produces a skew angle and the skew angle rotates in the counter clockwise direction with the increase of  $\beta$ .  $f(\epsilon)$  influences the spatial-temporal structures of the second- and third-order rogue waves. If  $f(\epsilon) \neq 0$ , the second-order rogue wave will split into three single first-order rogue waves and the triangular pattern can be formed, while the third-order rogue wave will split into six ones and the triangular pattern and pentagon pattern can be formed. The linear stability analysis is carried out, which shows that the modulation instability process is influenced by the amplitude of the harmonic wave and the wave number.

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## 1. Introduction

Interest in the rogue waves (also known as the freak, giant or extreme waves) has increased in recent years [1–12]. Rogue waves are localized in both space and time, appear from nowhere and disappear without a trace [5]. They have been seen in fluids [6], optical fibers [1,7,8], Bose-Einstein condensates [9,10], plasmas [11,12] and so on.

Recent theoretical and experimental studies have confirmed the possibility of switching the magnetization direction in the small magnetic domains by pumping the large spin-polarized currents [13–22]. From the application point of view, the current-induced magnetization switching has opened a way to control and manipulate the magnetization that is one of the issues in the magnetic technologies [13]. Phenomenon of the magnetization switching which attributes to the spin torque has been proposed [23,24]. Experimental verification of the spin torque has been carried out in the magnetic nanowires [25,26], spin valve pillar structures [19,21,27] and magnetic tunnel junctions [28,29].

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In Refs. [30–32], an infinitely-long ferromagnetic nanowire has been considered, where the current flows along the length of the nanowire, defined as the  $x'$  direction. The  $z$  axis has been taken as the direction of the anisotropy<sup>1</sup> field and the external field, and that condition can be experimentally realized [30–32,34]. It has also been assumed that the magnetization is nonuniform only in the  $x'$  direction [30–32,34]. The current injected in the nanowire has been shown to be polarized and produce a spin torque acting on the localized magnetization, which is written as  $\tau_b = b_j \partial \mathbf{M} / \partial x'$ , where  $\tau_b$  denotes the spin torque vector,  $\mathbf{M} = \mathbf{M}(x', t')$  is the localized magnetization vector with  $t'$  as the time, the spin-torque parameter  $b_j = P j_e \mu_B / (e M_s)$  has a dimension of the velocity,  $P$  is the spin polarization of the current,  $j_e$  is the current density,  $\mu_B$  is the Bohr magneton,  $e$  is the magnitude of electron charge and  $M_s$  is the saturation magnetization [13,30–32,34].

Dynamics of the localized magnetization with spin torque can be described by the modified Landau–Lifshitz equation including a term for the spin torque [30–32,34–37]. In this paper, we will focus on a particular type of the modified Landau–Lifshitz equation, i.e., Eq. (4) which will be introduced in Section 2. To our knowledge, conservation laws, linear stability analysis and higher-order rogue waves for Eq. (4) have not been reported in the existing literatures. In Section 2, the mathematical description of the localized magnetization with spin torque will be given, and Eq. (4) will be derived under the long-wavelength approximation. In Section 3, Lax pair and conservation laws will be constructed. In Section 4, generalized Darboux transformation (DT) will be derived. Rogue wave solutions will be obtained via generalized DT in Section 5. Property of the rogue waves and influence of the related parameters on the rogue waves will also be discussed. In Section 6, the linear stability analysis will be carried out. The last section will be the conclusion.

## 2. Dynamics of localized magnetization with spin torque

The localized magnetization has been shown to be described by the modified Landau–Lifshitz equation including a term for the spin torque [30–32,34–37],

$$\frac{\partial \mathbf{M}}{\partial t'} = -\gamma \mathbf{M} \times \mathbf{H}_{\text{eff}} + \frac{\rho}{M_s} \mathbf{M} \times \frac{\partial \mathbf{M}}{\partial t'} + \tau_b, \quad (1)$$

where  $\gamma$  is the gyromagnetic ratio,  $\rho$  is the Gilbert damping parameter, and  $\mathbf{H}_{\text{eff}}$  denotes the effective magnetic field which can be expressed as [30–32,34]

$$\mathbf{H}_{\text{eff}} = \frac{2A}{M_s^2} \frac{\partial^2 \mathbf{M}}{\partial x'^2} + \left[ \left( \frac{H_K}{M_s} - 4\pi \right) M_z + H_{\text{ext}} \right] \mathbf{e}_z, \quad (2)$$

with  $A$  as the exchange constant,  $H_K$  as the anisotropy field,  $H_{\text{ext}}$  as the external field,  $M_z$  as the demagnetization field, and  $\mathbf{e}_z$  as the unit vector along the  $z$  direction. Introducing the normalized magnetization vector  $\mathbf{m} = \mathbf{M}/M_s$ , people have written Eq. (1) in the following dimensionless form [30–32],

$$\frac{\partial \mathbf{m}}{\partial t} = -\mathbf{m} \times \frac{\partial^2 \mathbf{m}}{\partial x^2} + \delta \mathbf{m} \times \frac{\partial \mathbf{m}}{\partial t} + \frac{b_j t_0}{l_0} \frac{\partial \mathbf{m}}{\partial x} - \left( m_z + \frac{H_{\text{ext}}}{H_K - 4\pi M_s} \right) \mathbf{m} \times \mathbf{e}_z, \quad (3)$$

where  $t = t'/t_0$ ,  $x = x'/l_0$ ,  $t_0 = 1/[\gamma(H_K - 4\pi M_s)]$  represents the characteristic time,  $l_0 = \sqrt{2A/[M_s(H_K - 4\pi M_s)]}$  is the characteristic length, while  $\mathbf{m}(x, t) = (m_x(x, t), m_y(x, t), m_z(x, t)) = (0, 0, 1)$  forms the ground state for the normalized magnetization with  $m_x(x, t)$ ,  $m_y(x, t)$  and  $m_z(x, t)$  as the  $x$ ,  $y$  and  $z$  components of  $\mathbf{m}(x, t)$ , respectively [30]. When the magnitude of the magnetic field is high enough, the deviation of the magnetization from the ground state has been claimed to be small, so that  $m_z \approx 1$ , and the transformation  $\psi(x, t) = m_x + i m_y$  with  $i^2 = -1$  has been used [30]. When people consider the case without the damping under the long-wavelength approximation [38], and take only the nonlinear terms of the order of the magnitude of  $|\psi|^2 \psi$ , Eq. (3) has been shown to be transformed into the following integrable equation about  $\psi$  [30–32,39],

$$i \psi_t = \psi_{xx} + \frac{1}{2} |\psi|^2 \psi - \left( 1 + \frac{H_{\text{ext}}}{H_K - 4\pi M_s} \right) \psi + i \frac{b_j t_0}{l_0} \psi_x. \quad (4)$$

For Eq. (4), soliton solutions with the ground state as the seed solution have been obtained via the Hirota method [31,39]. With the help of DT, soliton solutions for Eq. (4) with the nonlinear spin wave as the seed solution have been derived [30]. Formation of the magnetic rogue waves for Eq. (4) has been investigated, which shows that the accumulation of energy and magnons<sup>2</sup> plays the main role for the generation of the magnetic rogue waves [32].

## 3. Lax pair and conservation laws

In order to obtain the rogue wave solutions, we derive a Lax pair for Eq. (4), which is different from that obtained in Ref. [30], by employing the Ablowitz–Kaup–Newell–Segur formalism [42],

$$\Psi_t = U \Psi, \quad (5a)$$

<sup>1</sup> Magnetic anisotropy is the directional dependence of a material's magnetic property. The magnetic moment tends to align with an easy axis, which is an energetically favorable direction of the spontaneous magnetization [33].

<sup>2</sup> Magnon can be viewed as a quantized spin wave which carries a fixed amount of energy and lattice momentum [40,41].

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