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Research paper

Solitary and Jacobi elliptic wave solutions of the generalized Benjamin-Bona-Mahony equation

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ABSTRACT

Exact bright, dark, antikink solitary waves and Jacobi elliptic function solutions of the generalized Benjamin-Bona-Mahony equation with arbitrary power-law nonlinearity will be constructed in this work. The method used to carry out the integration is the F-expansion method. Solutions obtained have fractional and integer negative or positive power-law nonlinearities. These solutions have many free parameters such that they may be used to simulate many experimental situations, and to precisely control the dynamics of the system.

1. Introduction

The search of exact solutions for evolution nonlinear partial differential equations (NLPDEs) plays an important role in the study of nonlinear physical phenomena. This is due to the fact that, nonlinear phenomena are ubiquitous in nature, thus appear in a wide range of fields in physics, mathematical physics, and engineering. Some celebrated evolution NLPDEs are the Hasegawa–Mima equation [1] which describes turbulence in plasma physics, the Fitzhugh–Nagumo equation [2] that models biological neuron, the Hunter–Saxton equation [3,4] used to study waves orientation in nematic liquid crystal, the nonlinear Schrödinger equation that models the dynamics of waves in many media such as matter waves in Bose–Einstein condensates [5], the evolution of electromagnetic fields in fiber optics [6], the evolution of gravity driven surface water waves [7], the evolution of the order parameter in the BCS theory [8], just to name a few.

In the past five decades, a great deal of attention has been paid to the dynamics of shallow water waves, mainly modeled by an evolution NLPDE known as the Korteweg-de Vries (KdV) equation [9], and modified KdV equations [9]. The KdV equation is valid when the water depth is constant and is derived under the assumption of small wave-amplitude and large wave length. Modified KdV equations include KdV equations with varying bottom and higher order corrections of the KdV equation. Solutions of the KdV and modified KdV equations have actively been investigated, and include solitary waves which come from a delicate balance between dispersion and nonlinearity, periodic waves like the Jacobi elliptic function solutions and so on [9,10].

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In 1972, the regularized long-wave equation, better known as the Benjamin-Bona-Mahony (BBM) equation [11] were introduced as a regularized form of the KdV equation. As pointed out in [11,12], the BBM equation better describes long waves and, as far as the existence, uniqueness and stability are concerned, the BBM equation has some substantial advantages over the KdV equation [11]. Moreover, the BBM equation also finds applications in other contexts such as the modeling of the drift of waves in plasma physics or the Rossby waves in rotating fluids [13], wave transmission in semi-conductors and optical devices [14], hydromagnetic waves in cold plasma, acoustic waves in anharmonic crystals, and acoustic gravity waves in compressible fluids [11]. In the past four decades, the BBM equation and its various versions have been intensively studied, and many types of solutions have been found among them solitary waves and periodic waves which can be found in the explicit literature. A classification of some forms of the BBM equation and their solutions can be found in [15]. Unlike the KdV equation, the BBM equations are not exactly integrable in the sense of the Painlevé test of integrability. Nevertheless, various techniques have been developed which help to carry out the integration of BBM equations. Among them are the tanh and the sine-cosine methods [16], the Jacobi elliptic function expansion method [17], the first integral method [18], the variable-coefficient balancing-act method [19], the hyperbolic auxiliary function method [20], the homogeneous balance method [21]. A generalized (1+1) BBM equation with dual arbitrary power-law nonlinearity may be written in dimensionless form as [22,23]

$$u_t + \alpha u_x + (\beta u^n + \gamma u^{2n})u_x - \delta u_{xxt} = 0, \tag{1}$$

where u is the wave profile, α and δ are the dispersion coefficients, n is the arbitrary power-law nonlinearity, β and γ the coefficients of the dual power-law nonlinearity. For $\alpha = 1,\ \beta = 1,\ n = 1,\ \gamma = 0,$ and $\delta = 1$ Eq. (1) recovers the BBM equation. Wazwaz solved Eq. (1) in the case where $\alpha = 1,\ \delta = -1,\ \beta = 0$ by means of the tanh and the sine-cosine methods, and obtained solitary and periodic solutions [24]. Yang, Tang, and Qiao constructed solitary and periodic wave solutions of Eq. (1) for $\alpha = 0,\ n > 0$ and $\delta \neq 0$ using an improved tanh function method [22]. Liu, Tian, and Wu used the Weierstrass elliptic function method to construct two solutions of Eq. (1) for $\alpha = 0$ in terms of the Weierstrass elliptic functions [25]. Biswas employed the solitary wave ansatz method and proposed a one-soliton solution of Eq. (1) [23]. All the latter works show the importance of investigating solutions of the BBM equation given by Eq. (1). However, to the best of our knowledge, Eq. (1) with non-vanishing coefficients has only been tackled in the work of Ref. [23] where a one-soliton solution was found. As an evolution NLPDE with important applications in different fields in physics, it is important to find more solutions of Eq. (1) that may help to have a better understanding of physical phenomena or at least give orientations for future applications. For example, solitary and Jacobian elliptic function solutions have been intensively used for practical applications in physics and engineering; these solutions have not been fully investigated for the generalized BBM equation (Eq. (1)) with all non-vanishing coefficients.

The aim of this work is to construct solitary and Jacobi elliptic wave solutions of the generalized BBM equation (Eq. (1)) with all non-vanishing coefficients. To this end, we use the F-expansion method introduced in [26] which has been an accurate tool to integrate evolution NLPDEs, along with the auxiliary ordinary equation [5,27]. The paper is organized as follows, in Section 2, we construct analytical solutions of Eq. (1). Then we discuss the characteristics and evolution of the solutions in Section 3. The paper is concluded in Section 4.

2. Analytical solutions

We start our quest of analytical solutions of Eq. (1) by setting the following traveling wave transformation

$$u(x,t) = U(\zeta)\,,\ \zeta = kx - Vt\,, \tag{2}$$

where k is the inverse of the width of the wave and V its velocity. Inserting Eq. (2) into Eq. (1) we obtain a nonlinear ordinary differential equation for the function U

$$(\alpha k - V)U_\zeta + k(\beta U^n + \gamma U^{2n})U_\zeta + \delta k^2 V U_{\zeta\zeta\zeta} = 0\,, \tag{3}$$

with $U_\zeta \equiv \frac{\partial U}{\partial \zeta}$ and $U_{\zeta\zeta\zeta} \equiv \frac{\partial^3 U}{\partial \zeta^3}$. An integration of Eq. (3) yields

$$(\alpha k - V)U + k\left(\frac{\beta}{n+1}U^{n+1} + \frac{\gamma}{2n+1}U^{2n+1}\right) + \delta k^2 V U_{\zeta\zeta} = 0\,, \tag{4}$$

where the right-hand side constant of integration has been set to zero. Eq. (4) is difficult to solve analytically, in order to find analytical solutions, we need to transform it into a more tractable and manageable form. Toward that end, we use the transformation $\omega = U^n$. After a little algebra, a nonlinear ordinary differential equation in terms of the function ω is retrieved

$$\omega\omega_{\zeta\zeta} + p\omega^2 + q\omega^3 + r\omega^4 + s(\omega_\zeta)^2 = 0\,, \tag{5}$$

in which the parameters p, q, r, s are given by

$$p = \frac{(\alpha k - V)n}{\delta k^2 V}\,, \tag{6a}$$