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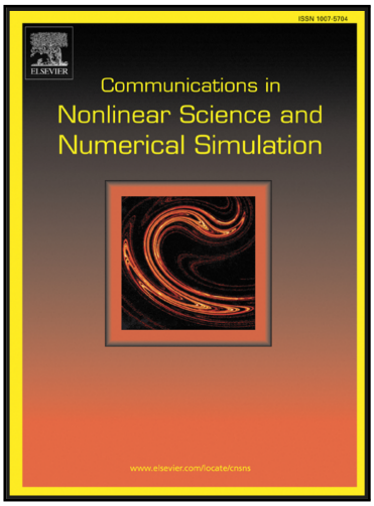

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ANALYSIS OF A CLASS OF BOUNDARY VALUE PROBLEMS DEPENDING ON LEFT AND RIGHT CAPUTO FRACTIONAL DERIVATIVES

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Abstract. In this work we study boundary value problems associated to a nonlinear fractional ordinary differential equation involving left and right Caputo derivatives. We discuss the regularity of the solutions of such problems and, in particular, give precise necessary conditions so that the solutions are $C^1([0,1])$. Taking into account our analytical results, we address the numerical solution of those problems by the *augmented*-RBF method. Several examples illustrate the good performance of the numerical method.

Primary 26A33, 34K10; Secondary 33C05, 34L30.
Fractional Calculus, Caputo Derivative, boundary value problem, radial basis functions

1. Introduction

The second order differential equation $y'' = f(t,y)$ is widely studied in the literature, whether one looks for analytical or numerical/computational results.

The fractional calculus (cf. Section 2 for basic definitions and results) is nowadays a topic of intensive research (see [8, 16] and the references therein). In the fractional calculus theory the intuitive way of generalizing the previous differential equation is substituting the classical operator y'' by a fractional one, say, the Caputo fractional derivative of order $1 < \alpha \leq 2$, ${}_0^C D^\alpha y$, i.e. to consider the following fractional differential equation ${}_0^C D^\alpha y = f(t,y)$.

A not so obvious, yet possible, way to generalize the second order differential equation is to consider the following fractional ordinary differential equation (FODE):

$$(1) \qquad {}^C D_1^\beta {}_0^C D^\alpha y(t) = f(t, y(t)), \quad t \in [0,1], \quad 0 < \alpha, \beta \leq 1.$$

(usually this equation will be subjected to some boundary conditions). Perhaps the main reason to use the left and right fractional differential operators as in (1) is the resemblance of equation in (1) with the Euler–Lagrange equation that arises from fractional calculus of variations problems. Indeed, if one consider the following variational problem,

$$J(y) = \int_0^1 L(t, y(t), {}_0^C D^\alpha y(t))dt \to \min,$$

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