Computers and Fluids 159 (2017) 23–32

Contents lists available at ScienceDirect

Computers and Fluids

journal homepage: www.elsevier.com/locate/compfluid

Cholesky decomposition–based generation of artificial inflow turbulence including scalar fluctuation

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ARTICLE INFO

Article history:
Received 26 November 2016
Revised 24 August 2017
Accepted 9 September 2017
Available online 11 September 2017

Keywords:
Artificial turbulence generation
Inflow boundary condition
Large-eddy simulation
Space correlation
Time correlation
Cholesky decomposition

ABSTRACT

This paper proposes a new method for generating turbulent fluctuations in wind velocity and scalars, such as temperature and contaminant concentration, based on a Cholesky decomposition of the time-averaged turbulent flux tensors of the momentum and the scalar for inflow boundary condition of large-eddy simulation (LES). The artificial turbulent fluctuations generated by this method satisfy not only the prescribed profiles for the turbulent fluxes of the momentum and the scalar but also the prescribed spatial and time correlations. Based on an existing method that is able to impose the spatial and time correlations using digital filters, random two-dimensional data are filtered to generate a set of two-dimensional data with the prescribed spatial correlation. Then, these data are combined with those from the previous time step by using two weighting factors based on an exponential function. The method was validated by applying generated inflow turbulence to an LES computation of contaminant dispersion in a half-channel flow.

1. Introduction

Computational fluid dynamics (CFD) is widely used in the field of computational wind engineering to predict urban wind environments [1,2]. Most attempts have utilized turbulence models based on the Reynolds-averaged Navier–Stokes equations [1,3–5]. In recent years, the growth of computing power makes it possible to apply large-eddy simulation (LES) to urban areas. One of the biggest unresolved issues involves the generation of inflow turbulence that reproduces upstream flow conditions. It is known that inflow turbulence properties greatly affect the flow field around buildings [6–8]. This means that time-dependent inflow turbulence within the atmospheric boundary layer should be well reproduced not only for the turbulence statistics but also for the instantaneous turbulent structures.

Various inflow generating methods have been proposed, as reviewed in detail by Tabor and Ahamadi [9] and Wu [10]. According to Wu [10], these techniques for generating inflow turbulence can be classified into two categories: recycling methods and synthetic methods. Recycling methods consist of strong recycling methods, which are based on recycling simulation [11,12], and weak recycling methods, which are based on rescaling–recycling simulation [13,14]. Recycling methods store the time history of velocity fluctuations obtained from a recycling CFD computation conducted as a preliminary simulation. The weak recycling methods proposed by Lund et al. [14] and Kataoka and Mizuno [15] have been widely used to generate fully developed boundary layers used as inflow boundary condition for LES around buildings.

In contrast, synthetic methods artificially generate inflow turbulence with prescribed turbulence statistics using random numbers without conducting CFD computations. An advantage of the synthetic methods is that they do not require preliminary simulation of the flow field to obtain the fluctuations used as the inflow boundary condition of the main simulation. Hence, the total computational cost is reduced compared to recycling methods. As shown by Wu [10], the synthetic methods mainly consist of four approaches: (1) synthetic random Fourier methods (SRFMs), which are based on inverse Fourier transforms of prescribed power spectra and cross spectra [16–19], (2) synthetic digital filter methods (SDFMs), which are based on the convolution of random numbers with a spatial digital-filter [20–23], (3) synthetic coherent eddy methods (SCEMs), which are based on the superposition of a large number of convected eddies [24,25], and (4) synthetic volume forcing methods (SVFMs), which are based on the addition of synthetic body force terms to the Navier–Stokes equations [26,27].

SRFM is a pioneering technique for synthetic methods. Lee et al. [16] proposed an SRFM that uses the inverse Fourier transform and satisfied the prescribed power spectra for homogeneous isotopic

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http://dx.doi.org/10.1016/j.compfluid.2017.09.005

turbulence. Then, a spatially decaying turbulence simulation was performed using the artificially generated homogeneous isotopic turbulence. Because the synthetic fluctuations are calculated independently, the Reynolds stress will be zero. Le and Moin [28,29] revised Lee et al.'s SRFM [16] for wall-bounded flows in which the distributions of Reynolds stress should be considered based on a coordinate transformation of the generated synthetic fluctuations, and performed a direct numerical simulation (DNS) of the flow field over a backward-facing step with generated anisotropic turbulence as the inflow boundary condition. The summary of the transformation with Cholesky decomposition of the Reynolds stresses tensor matrix is introduced in the appendix of Lund et al. [14] and briefly described below. The values of wind velocity, u_i, are divided into the time-averaged values of u_i, $\langle u_i \rangle$, and the deviation from the time-averaged value, $u_i{}'$:

$$u_i = \langle u_i \rangle + u_i'. \tag{1}$$

With the lower triangular matrix of the Reynolds stress tensor matrix, a_{ij}, and a variable, Ψ_j, satisfying $\langle \Psi_j \rangle = 0$ and $\langle \Psi_i \Psi_j \rangle = \delta_{ij}$, instantaneous wind velocity, u_i, that mathematically satisfy the prescribed Reynolds stresses can be expressed as

$$u_i = \langle u_i \rangle + u_i' = \langle u_i \rangle + a_{ij} \Psi_j. \tag{2}$$

The mathematical theory suggests any Ψ_j can be chosen so long as it satisfies $\langle \Psi_j \rangle = 0$ and $\langle \Psi_i \Psi_j \rangle = \delta_{ij}$. This transformation was immediately recognized. Then, SDFMs and SCEMs were developed to generate Ψ_j satisfying not only the prescribed Reynolds stresses but also spatial and time correlations.

Jarrin et al. [24] developed an SCEM that calculates Ψ_j as the superposition of a large number of eddies within a length scale related to the turbulence integral length scale. Klein et al. [20] proposed an SDFM in which Ψ_j are defined as a three dimensional convolution of random numbers with a digital filter to impose space correlation. Xie and Castro [21] revised and simplified Klein et al.'s approach [20] to efficiently generate an inflow turbulence with a two dimensional digital filter and demonstrated LES over a building array. This method imposes space correlations on the wind velocity fluctuations of the lateral and vertical directions and time correlations on those of the streamwise direction.

Almost simultaneously, Spille-Kohoff and Kaltenbach [26] proposed an SVFM and demonstrated a spatially developing turbulent boundary layer similar to that of Lund et al. [14]. In this method, direct forcing terms added in the Navier–Stokes equations are applied in a control zone adjacent to the inlet boundary to achieve a prescribed Reynolds stress distribution. The instantaneous body force is computed with a PI controller as a function of the error signal, which is the difference between the current time step and the prescribed values.

In recent years, several non-isothermal LES computations within urban boundary layers were performed [30–33]. When LES is applied to a non-isothermal flow-field, both the inflow velocity fluctuation and the air temperature fluctuation should be reproduced. Only a few inflow generation methods for wind velocity and air temperature fluctuations have been proposed.

Kong et al. [34] proposed a generation method for inflow fluctuations of wind velocity and air temperature using Lund et al.'s method [14] according to the same rescaling and recycling law between air temperature and streamwise velocity. Hattori and Nagano [35] conducted DNSs for very weak stable and unstable boundary layers with the inflow turbulent fluctuations of wind velocity and air temperature generated by the method proposed by Kong et al. [34]. Jiang et al. [36] carried out LESs for weak stable and unstable boundary layers with the inflow turbulence including air temperature fluctuation generated by extending Kataoka's weak recycling method [15]. Tamura et al. [31] conducted a non-isothermal LES of the Tokyo metropolitan area with an inflow turbulence including air temperature fluctuation. They set up a driver region that consists of two domains in the preliminary calculation to generate inflow turbulence with the temperature fluctuation. In the first domain, a fully developed isothermal boundary layer over a rough wall was calculated with the recycling method by Lund et al. [14]. Then, in the second domain, a stable boundary layer simulated as a sea breeze was calculated using turbulent fluctuations stored from the calculation in the first domain as the inflow boundary condition, and time histories of the wind and the temperature fluctuations at the outlet boundary were stored for the main simulation. Yoshie et al. [30] performed a preliminary LES in which the approaching section of heated roughness elements in a wind tunnel was completely reproduced and predicted gas dispersion around a high-rise building under an unstable stratification condition. Mirocha et al. [37] proposed a synthetic inflow-generation-method for nested LES from meso-scale flow predicted with a meso-scale meteorological model based on the concept of SVFM because the meso–scale flow includes essentially no resolved turbulent fluctuations. Forcing terms were added to the momentum equations for the horizontal directions and the transport equation for potential temperature to provide minimal disturbance to the flow field so turbulence could develop naturally within the nested LES domain near the inflow boundaries. Muñoz-Esparza et al. [38,39] developed several perturbation generation methods for potential temperature with the forcing method to accelerate the generation of turbulence on nested LES inflow boundaries.

However, generation methods for wind velocity and air temperature fluctuations based on preliminary or subsequent calculations require additional computational costs. Furthermore, in these methods, the distributions of mean wind velocity and air temperature and their fluctuations are obtained after the preliminary computation. This means it is difficult to impose the prescribed profiles of wind velocity and air temperature and the turbulence fluxes of the momentum and air temperature. Synthetic methods using forcing terms require optimization of the location of the control zone and the parameters in the forcing terms. Thus, the process of the trial and error is usually needed for the current inflow generation methods.

This paper proposes a new method of artificially generating turbulent fluctuations in wind velocity and a scalar, such as air temperature and concentration, based on a simultaneous Cholesky decomposition of the time-averaged turbulence flux tensors of the momentum and the scalar. The artificial turbulent fluctuations generated using this method satisfy not only the prescribed profiles for the turbulence fluxes of the momentum and the scalar but also the prescribed spatial and time correlations. According to the method proposed by Xie and Castro [21], two-dimensional random data are filtered to generate a set of two-dimensional data with the prescribed spatial correlation. Then, these data are combined with those from the previous time step by using two weighting factors based on an exponential function corresponding to a time correlation of the fluctuations. The method was validated by applying the generated turbulent fluctuations to the inflow boundary conditions of an LES computation of contaminant dispersion without buoyancy in a half-channel flow.

2. New method of generating inflow turbulence including scalar fluctuation

In this study, we express the values of wind velocity and scalar as f_i, the time-averaged values of f_i as $\langle f_i \rangle$, and the deviation from the time-averaged value as $f_i{}'$:

$$f_i = \langle f_i \rangle + f_i', \tag{3}$$