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Benchmark solutions

A three-dimensional source-vorticity method for simulating incompressible potential flows around a deforming body without the Kutta condition

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ABSTRACT

For predicting three-dimensional incompressible potential flows around a body accompanied by a wake vortex, surface singularity methods (i.e., panel methods) have been employed extensively, owing to their ease of use and low solution times. In the case of lifting/vortical flow, the Kutta condition is applied, in order to insure smooth flow at the trailing edge. However, the Kutta condition is inapplicable in the case of blunt bodies. For this reason, a three-dimensional source-vorticity method for simulating incompressible potential flows around a deforming body without using the Kutta condition is presented. For lifting/vortical flows, three components of the surface vorticity vectors are placed on the panels instead of the doublet as the unknowns. In place of the Kutta condition, additional equations are employed for determining the total circulations for the three vorticity components about the body. To validate the proposed method, simple examples, such as a sphere in a uniform flow and a sphere in an accelerated flow, are treated as non-lifting/non-vortical cases. For lifting/vortical cases where the Kutta condition cannot be applied, a rotating sphere in a uniform flow and a sphere with a vortex ring are considered. To assess the accuracy of the proposed method, the numerical results are compared with the analytical solutions. Finally, to highlight the applicability of the method in the case of unsteady lifting/vortical flow and to show its versatility as well as suitability in treating deforming bodies, a swimming great white shark is simulated with and without the wake vortex. Based on the results obtained in the absence of the wake vortex, it was found that, even in an inviscid flow, a thrust force is produced by the movement of the shark. Further, the results obtained for the case where a wake vortex was shed from the tail fin suggested that the wake vortex sheets decrease the amplitude of the side force and increase that of the thrust force.

1. Introduction

For predicting three-dimensional (3D) incompressible potential flows around a body accompanied by a wake vortex, surface singularity methods (i.e., panel methods) have been employed extensively, owing to their ease of use and low solution times [1–6]. In these methods, the surface of the body in the fluid is discretized into triangular or tetragonal panels. The distributions of the source and/or the doublet are placed as unknowns on the panels to recreate flows both on the body surface and in the fluid around the body. The wake vortex is expressed by the doublet [1–4] or the vorticity [5–6]. By enforcing the normal-velocity boundary condition on the body surface, these distributions can be determined uniquely, so that the stream lines follow the body surface and correspond to the solutions of the incompressible potential flow.

Further, in the case of lifting/vortical flow, the Kutta condition is applied, in order to ensure smooth flow at the trailing edge as well as to determine both the potential jump at the trailing edge and the circulation about the wake vortex. However, the Kutta condition is inapplicable in the case of blunt bodies [1]. For instance, to take a simple lifting/vortical case corresponding to potential flows, a rotating sphere in a uniform flow cannot be treated in this manner, because there is no trailing edge for the Kutta condition to be applied to.

For two-dimensional flow around a circular cylinder, an unsteady panel method using the Kutta condition was reported recently [6]. In this method, the separation location where the Kutta condition is applied is obtained from experimental data and is fixed in time. However, in actual flow, the separation location varies with time; thus, it is difficult to use the Kutta condition. On the other hand, an unsteady vortex method for a two-dimensional circular cylinder that does not use the Kutta condition was intro-

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duced by the author in 1991 [7]. The time variation in the separation location is successfully captured by this method.

Owing to this, a new approach for three-dimensional flows that does not use the Kutta condition for predicting lifting/vortical flows is developed. For non-lifting/non-vortical flows, only the source distribution is used to represent the body, and it is determined by a set of simultaneous equations, as has been done in previous studies. However, for lifting/vortical flows, in addition to the source distribution mentioned above, three components of the surface vorticity vectors are placed on the panels as the unknowns and not the doublet, in contrast to the case in previous studies.

Based on the normal-velocity boundary condition, the three sets of simultaneous equations to determine the three vorticity components are derived. Furthermore, instead of the Kutta condition, additional equations are employed to determine the total circulations for the three vorticity components about the body. These three sets of simultaneous equations are solved independently. Mracek et al. [9] presented a method to represent a 3D body using the vorticity vectors for steady flows. However, their method has not been applied in the case of unsteady lifting/vortical flows.

To validate the proposed method, simple examples such as a sphere in a uniform flow, a sphere in an accelerated flow, and an accelerated sphere in a fluid at rest are treated as non-lifting/non-vortical cases. For lifting/vortical cases where the Kutta condition cannot be applied, a rotating sphere in a uniform flow and a sphere with a vortex ring are considered. In order to assess the accuracy of the proposed method, the numerical results are compared with the analytical solutions.

Since the proposed method does not directly treat the velocity potential as the unknown, it has to be calculated later. For this, we introduce a simple but efficient method; in this method, the velocity potential is not calculated by directly integrating the surface velocity but by solving a set of simultaneous equations.

On the other hand, there is considerable research interest in unsteady/biological propulsion. To study the performances of swimming fishes, various schemes based on Eulerian methods have been presented. However, most of these studies have been limited to two-dimensional (2D) models of the swimming fishes [10–12]. In the case of 3D simulations, the fish shape has been kept fixed over time [13,14]. This is because Eulerian methods require that a new mesh system be used around the deforming body at every time step, which is not an easy task. One of the advantages of the surface singularity methods over Eulerian methods is that they do not require mesh generation around the body. Lastly, in order to highlight the applicability of the proposed method in the case of unsteady lifting/vortical flow as well as to show its versatility and suitability in treating deforming bodies, a swimming great white shark is simulated with and without the wake vortex.

2. Method

2.1. Non-lifting/non-vortical flows

In previous studies, for the cases where the fluid around the body does not exhibit vorticity and the body does not show circulation, only the source distribution has been used to represent the body. In this study, the 3D body is discretized into triangular panels. The velocity potential on the j-th panel produced by a source on the i-th panel is given by

$$\Phi(x_j, y_j, z_j) = \int_{S_i} \frac{\sigma_i}{4\pi r_{ij}} ds \tag{1}$$

where r_{ij} is the distance between the control points of the i-th and j-th panels, and σ_i is the strength of the source on the i-th panel. The control point of a panel is its centroid. The strength of the source is determined by solving the integral equation. The details

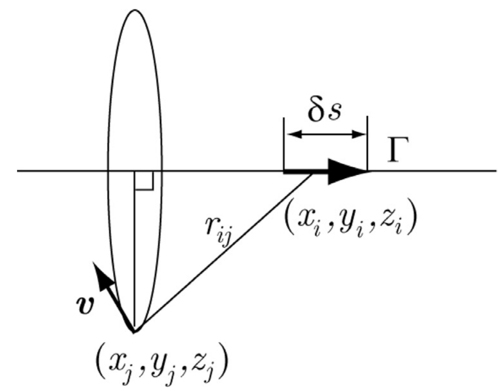

Fig. 1. Velocity vector $\boldsymbol{v}$ created by vortex filament of strength Γ and length δs.

of the method are not included as they have been explained previously in [1,8,15].

2.2. Lifting/vortical flows

When the fluid around the body exhibits vorticity or when the body has circulation, the three components of the vorticity vector bounded at the centroid of each panel on the body surface are used. For example, when a rotating sphere is placed in a uniform flow, the effect of the uniform flow around the sphere at rest is treated by the method given in Section 2.1 (called the source method). On the other hand, that of a rotating sphere in a fluid at rest such that the sphere has circulation is treated by the method described in this section (this method is called the vorticity method). The two solutions obtained by the methods given in Sections 2.1 and 2.2 are then combined to calculate the velocity field, pressure distribution, and forces.

First, we consider the x-component of the vorticity vector on the centroids of the panels on the body surface. The velocity vector, $\boldsymbol{v}_j^x$, created on the centroid of the j-th panel, (x_i, y_i, z_i), by the x-component of a vortex filament of the small vector (δs, 0, 0) with a total strength/circulation of $\Gamma_i^x \delta s$ and placed at the centroid of the i-th panel, (x_j, y_j, z_j), is given by [16]

$$\boldsymbol{v}_j^x = \sum_{i=1}^{N} \frac{\Gamma_i^x \delta s}{4\pi {r_{ij}}^3}\left(0, \; -z_j + z_i, y_j - y_i\right) = \sum_{i=1}^{N} G_i^x \boldsymbol{g}_{ij}^x \tag{2}$$

where Γ_i^x is the strength of the x-component of the vortex filament per unit length (Fig. 1). For simplicity, the total strength, $\Gamma_i^x \delta s$, is replaced by G_i^x. Further, the vector components $(0, -z_j + z_i, y_j - y_i)/(4\pi {r_{ij}}^3)$ are also replaced by a vector, $\boldsymbol{g}_{ij}^x$. The dimension of G_i^x is that of $\Gamma_i^x \delta s$, and $G_i^x (i = 1 \sim N)$ are treated as the unknowns to be determined.

The wake vortex in the fluid is treated as consisting of triangular or tetragonal vortex sheets that have a uniform vorticity on them. Using $\boldsymbol{W}_{kj}^x$ as the velocity vector created by the x-component of the k-th wake vortex sheet, the velocity vector, $\boldsymbol{v}_j^x$, and the normal-velocity boundary condition are given by

$$\boldsymbol{v}_j^x = \sum_{i=1}^{N} G_i^x \boldsymbol{g}_{ij}^x + \sum_{k=1}^{M} \boldsymbol{W}_{kj}^x \tag{3}$$

and

$$\begin{cases} \mathbf{n_j} \cdot \mathbf{v_j} = \mathbf{n_j} \cdot \left(\sum_{i=1}^{N} \mathbf{G_i^x g_{ij}^x} + \sum_{k=1}^{M} \mathbf{W_{kj}^x} \right) = \mathbf{0} \\ \sum_{i=1}^{N} \mathbf{G_i^x n_j} \cdot \mathbf{g_{ij}^x} = -\sum_{k=1}^{M} \mathbf{n_j} \cdot \mathbf{W_{kj}^x} \end{cases} \tag{4a}$$