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Smoothed particles hydrodynamics numerical simulations of droplets walking on viscous vibrating liquid

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ABSTRACT

We study the phenomenon of the "walking droplet", by means of numerical fluid dynamics simulations using the smoothed particle hydrodynamics numerical method. This phenomenon occurs when a millimetric drop is released on the surface of an oil of the same composition, contained in a tank and subjected to vertical oscillations of frequency and amplitude very close to the Faraday instability threshold. At appropriate values of the parameters of the system under study, the oil droplet jumps permanently on the surface of the vibrating liquid forming a localized wave-particle system, reminding the behaviour of a wave particle quantum system as suggested by de Broglie. In our study, we made relevant simplifying assumptions, however we observe that the wave-drop coupling is surely obtained. Moreover, the droplet and the wave travel at nearly constant velocity, as observed in experiments. These facts suggest that the phenomenon may occur in many contexts and opens the possibility to study it in an extremely wide range of physical configurations.

1. Introduction

The fact that a solid particle can jump persistently on a vibrating rigid desk is trivial. Less obvious is the similar occurrence when the particle is an oil droplet falling on a vibrating viscous liquid. The phenomenon of "walking droplets" has been discovered rather recently by Couder et al. [1] and it is currently studied by various researchers. The phenomenon has prompted much attention due to the analogy that this classical fact exhibits with the quantum mechanical behaviour of particles, e.g. see Couder and Fort [2]. It has been shown [1] that in an appropriated range of parameters, a droplet, deposited on a vibrating bath of a liquid of the same composition, can bounce indefinitely and generates a localized wave strongly connected to the droplet, which can move at constant horizontal velocity. To obtain this peculiar behaviour the amplitude of the vibration must be very close to the threshold of the Faraday instability. The phenomenon of the Faraday instability is well known since 1831 [3]: a viscous liquid, subjected to vertical oscillations of amplitude greater than a well-defined critical value correlated to the frequency of the oscillations, the surface tension, and to the viscosity of the liquid, produces standing waves with a frequency half of the driving one. A simplified theoretical analysis predicts that, in a shallow liquid with height of the order of few millimeters, the wavelength is expected to be, approximately, inversely proportional to the forcing frequency [4]. The requirement of working slightly below the Faraday instability threshold is due to the fact that, in this case, the instability produces waves only in proximity of the droplet collision zone and therefore a localized wave is generated, while the remaining liquid oscillates as a whole, but no wave is produced.

The particle-wave is affected by the boundary conditions and it may produce interference patterns and other typical quantum-like effects [5,6]. It is a classical analogue of the de Broglie idea of the wave–particle dualism occurring on atomic and subatomic scales.

The interest on reproducing this phenomenon by a numerical simulation is relevant. Although Milewski et al. [7] proposed tentative analytical equations for the motion of the individual drop and integrate them numerically taking into account the surface deformation, up to date, there are no truly fluid numerical simulation of this phenomenon. With affordable simulations, it is possible to investigate problems and configurations which are difficult to set up in laboratory. For example, it will be easily possible to investigate the role of an attractive force between two particles, like an elementary atom, or to make a linear oscillator and investigate the possibility of discrete energy levels, like the quantum oscillator. We will not discuss furthermore the quantum analogy, that, in our opinion, is far to be demonstrated (see also the arguments against the analogy by Andersen et al. [8]).

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In principle, the walking droplet problem is not very difficult to be studied with a Lagrangian numerical fluid dynamical method. The presence of waves deforming the liquid surface makes the Lagrangian codes better suited than the Eulerian ones. We studied this phenomenon using the smoothed particles hydrodynamics (SPH) numerical method to simulate the motion of the oil in the vessel, while the droplet is simulated by a small group of points (or even a single point, i.e. with no internal structure) each interacting both with the other droplet points and with the discrete particles of the oil by *ad hoc* forces described below.

A more realistic simulation would require the treatment also of the air dynamics and not only of the oil. This approach had to manage also the large density difference between air and liquid. This kind of problems have been studied with the SPH methodology [9]. However, this full physical simulation, even if it is possible, has some practical drawbacks. To have a realistic simulation of the surface tension, the simulated droplet should have a spatial resolution such to guarantee a number of SPH particles of the order, say, of 500 particles [10], then the total number of particles (air, oil and droplet) is consequently rather large for a 2D (X–Z) case. Indeed, if we assume that a droplet has a 0.5 mm radius, and that it should be composed by 500 particles, we need a spatial resolution of about 0.00004 m, then, to reproduce a container of length 0.1 m and vertical height 0.01 m, including air, we need about 636,000 particles. Furthermore, the time step falls down since the dt due to the artificial viscosity scales as h^2 and for any kind of viscosity the diffusion condition requires $dt \leq \frac{h^2}{\sigma_{visc}}$ [11]. For this reason, we preferred to avoid the simulation of the air component, to mimic the interaction of the droplet with air and to take into account the surface tension by some *ad hoc* simple and successful physical models described below. The possibility to obtain significant results with a modest number of particles is, in our opinion, an added value, which can be exploited without special computational resources.

We set up a small 2D tank of viscous liquid, vibrating very close to the Faraday resonance frequency. A droplet is released on the liquid surface and, with some simplifying assumptions, we show that the basic phenomenon of the wave particle coupling is reproduced.

Many ingredients concur to produce this phenomenon. We tried to mimic the basic ones. Despite we made some strong approximations to make feasible the simulations, the fact that we reproduce the general experimental behaviour indicates that some parameters are more essential than other ones and that the phenomenon may be physically more ubiquitous.

We emphasize that the aim of this communication is basically to show that with a quite simplified model and a numerical simulation based on SPH method it is possible to study this interesting phenomenon.

The subject is treated as follows: in Section 2 we discuss the problem and its approximations; in Section 3 we briefly explain the SPH numerical method; in Section 4 we discuss the parameters of the systems that we simulate and show some of the main results. Finally, in Section 5, general conclusions are summarized.

2. The model and its approximations

The basic physical elements required to produce the phenomenon are: (1) vibration of the tank at a frequency very close to the Faraday instability acting as energy reservoir and pump. (2) the specific amplitude of the oscillation, which can even produce accelerations larger than the gravitational one. (3) the viscosity of the liquid, to stabilize the dynamics. (4) the surface tension on the liquid and of the droplet to produce or enhance the bouncing force activated by the extra air pressure. (5) the air cushion to produce the bouncing force.

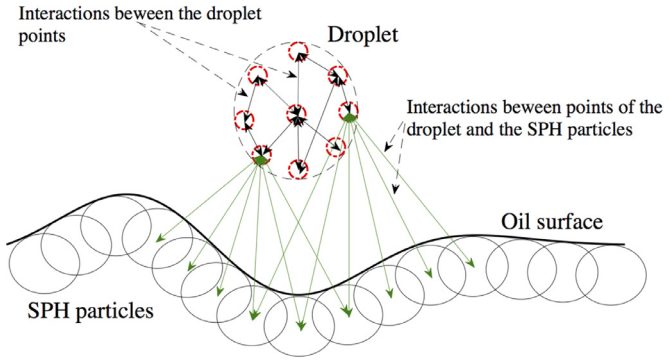

Fig. 1. Schematic view of the interactions among the droplet points and between them and the SPH liquid particles. The interactions between the SPH particles are not shown.

The interaction of this localized wave with the droplet at a specific vibration frequency, amplitude, together with a specific size of the droplet, produces the coupled wave-particle synchronized motion.

In our approach, we essentially studied two models of the droplet: one, a bit more realistic, in which the liquid droplet has a finite dimension, and it is made by a small group of points, held together by an appropriate small range force that produces a tension, and interacting with the liquid particles with another suitable force.

In a second model, extremely simple, the droplet is build up by a single point particle, i. e. a moving point, without any internal structure, interacting with the other liquid particles by an *ad hoc* force.

2.1. Interactions for the droplet made by points

2.1.1. The surface tension

With appropriate mathematical interpolation criteria for the boundaries, the SPH method can treat the surface tension of a liquid [12,10]. Obviously, these approaches require a number of interpolating points sufficiently large to accurately calculate the curvature of the surface and all the related derivatives for the liquid under study. As mentioned in the previous section, in the phenomenon that we want to study the liquid droplet interacts with the liquid in the tank by the presence of the air, but we decided to avoid the complexities of the two fluid treatment (liquid–air) and preferred to adopt an *ad hoc* strategy: while the liquid in the vessel will be treated by standard SPH, the droplet will be treated as a set of points, whose interaction with themselves in the droplet will provide the surface tension, while the interaction with the liquid in the vessel is realized with another kind of force acting between each point and the SPH particle of the liquid in the vessel. Fig. 1 gives a sketch of the proposed interactions treatment.

The “small group” model attributes to a single point, constituting a droplet component, an elastic force acting on all other droplet points. It allows droplet deformation and produces a fictitious tension, whose value, in this study, is not compared with the real one.

The force between point i and point j, we name $\vec{T}_{i,j}$, is given by the following expression:

$$\vec{T}_{i,j} = -\frac{f}{n_{drop}^2}\left(\frac{r_{i,j} - \delta_{drop}}{\delta_{drop}}\right)\frac{\vec{r}_{i,j}}{r_{i,j}} \tag{1}$$

where r_{ij} is the distance between two points, i and j, in the droplet, δ_{drop} is the size of the spatial distribution of the initial regular array of the points in the droplet, n_{drop} is the total number of points in the droplet, the intensity factor f is given by $f = 0.1c_0^2\rho_0 2s$,