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Numerical study on secondary flows of viscoelastic fluids in straight ducts: Origin analysis and parametric effects

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ABSTRACT

Numerical simulations were conducted on viscoelastic fluid flows in straight ducts with different cross sections, for which the origin of secondary flows and influences of material parameters and flow passage geometrical configuration were numerically investigated. The Giesekus constitutive model was chosen to describe the viscoelastic fluid with the second normal stress difference N_2, and solved by embedding UDF (User-defined Function) into the CFD (computational fluid dynamics) code FLUENT. The origin of such kind of secondary flow was theoretically studied from the perspective of the budget of vorticity energy for the first time. Sufficient and necessary condition for the existence of secondary flow was then developed in terms of N_2, the gradient of N_2 and cross-sectional geometry ϑ (i.e., generation term E_Ω). Moreover, helicity density was considered as an excellent indicator of secondary flow pattern. Effects of material properties (including anisotropic parameterα, solvent viscosity ratio β and relaxation time λ) and flow passage geometrical configuration (aspect ratio of cross sections$r$, the number of polygon sides n) on secondary flow strength and pattern were investigated with E_Ω. Finally, a universal variable σ_s was proposed to describe non-circularity of cross section, based on which the results for ducts with different cross sections can be normalized together.

1. Introduction

Viscoelastic fluid, a typical class of Non-Newtonian fluid in which viscosity and elasticity coexist, has received considerable attention for its wide practical applications in liquid transportation through pipes such as petroleum transportation, food production, polymer processing and so on. Plenty of experiments have been conducted on viscoelastic fluids where many unique and interesting physical phenomena (e.g., rod-climbing effect, turbulent drag reduction [1–6], Barus effect, unusual particle migration [7]) have been observed comparing with those in Newtonian fluids [8]. In general, an apparent difference between the rheological properties of viscoelastic and Newtonian fluids lies in the existence of normal stress differences, which is responsible for the origin of some peculiar flow behavior in viscoelastic fluids. For example, secondary flow, which exactly is the focus of this paper, would occur when viscoelastic fluid passes through a non-circular straight duct even in laminar regime, whereas no secondary flows occur in Newtonian fluid flow. Secondary flow in laminar viscoelastic fluid flow and turbulent Newtonian fluid flow share similar patterns but with inverse rotational directions [9]. The opposite rotational directions can be attributed to distinct extra lateral forces: the former is caused by elasticity, while the latter Reynolds stress. Referring to the similarity mentioned above, a kind of non-linear turbulent Reynolds stress model is inherited by elastic stress of viscoelastic fluid [10].

So far, there have been numerous researches concerning how the elasticity contributes to the existence of laminar secondary flow of viscoelastic fluid when passing through a non-circular straight duct. Ericksen [11] firstly mentioned that only when the cross section is circular or the apparent viscosity and normal stress functions satisfy certain relationships, would no secondary flow occur in steady flow of Reiner-Rivlin fluid through a straight duct. Then, Oldroyd [12] indicated the second normal stress difference (N_2) was tied with the generation of secondary flow in laminar viscoelastic fluid flow. He pointed out that steady rectilinear flow of viscoelastic fluid would occur if any of the following three conditions are met: (1) N_2 is zero; (2) both apparent viscosity and

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normal stress difference coefficient are constant or (3) these two are in proportional relationship. In this manner, a non-zero N_2 is only a necessary but not sufficient condition for the presence of secondary flow. Experiments of viscoelastic fluid with obviously non-zero N_2 were then successively conducted in straight ducts with various cross sections [13–17]. Dodson [13] adopted two kinds of viscoelastic fluids (aqueous solution of polyacrylamide and a soap solution with higher elasticity) and visualized secondary flows in the square and rectangular ducts through an injected dye thread. This work provided valid experimental support of the contribution of normal stress differences to secondary flow. Townsend et al. [14] tested six different viscoelastic fluid flow systems with different values of N_2, and concluded that the direction of the streamlines could not be indicated by the sign of N_2. This work theoretically demonstrated that the origin of secondary flow was related to the third derivative of N_2. Speziale [18] and Huang and Rajagopal [19] deduced that if there exists secondary flow, non-zero $S_{yy} - S_{xx}$ or non-zero S_{xy} (**S** is extra stress tensor) is necessary. Their works indicated the certain frame-indifferent convected time rates gave rise to secondary flow, rather than specific viscoelastic fluid models. Moreover, Holmes et al. [20] modeled transient and steady viscoelastic secondary flows via OpenFOAM with multimode Giesekus and PPT models and the multimode Giesekus model was accurate to capture secondary flow behavior. Therefore, the capture of the viscoelastic secondary flow requires non-linear viscoelastic fluid models with strain rate history independent of coordinate systems and non-zero N_2 instead of those with zero N_2 (upper-convected Maxwell (UCM) [21], Oldroyd-B [22] and simplified Phan-Thien-Tanner (SPTT) [23,24] models). Meanwhile, it is also necessary to pay attention to the effects of different parameters in various viscoelastic fluid models [25–27]. Gao and Harnett [25] performed a numerical study on the secondary flow of a Reiner-Rivlin fluid in laminar regime through ducts with square and rectangular cross sections, and studied the influences of N_2, Reynolds number (Re) and aspect ratio on the magnitude of secondary flow. Xue et al. [26] investigated the secondary flow patterns in square and rectangular cross sections by adopting modified Phan-Thien-Tanner (MPTT) constitutive model with non-zero N_2. It was found that material parameters had dramatic influence on secondary flow strength instead of secondary flow pattern. Yue et al. [27] investigated secondary flow of Giesekus viscoelastic fluid in a non-circular duct, and developed two conditions for the existence of secondary flow: (1) non-linear dependence of N_2 on shear viscosity and (2) non-axisymmetric cross section geometry. They concluded that secondary flow was caused by non-conservative "body force" arising from N_2, but not from N_2 directly. Letelier and his colleagues [28,29,30] proposed two causes responsible for secondary flow of viscoelastic fluid in straight ducts: the non-circular cross section and the non-conservative body force. With the aforementioned works, there still lacks thorough theories explaining the mechanism of secondary flow in pressure-driven laminar viscoelastic fluid flow through straight ducts with non-circular cross sections so far. Both non-linear material behavior and geometrical confinement bring about the difficulty in clarifying the origin of secondary flow. It is necessary to find other feasible ways to explain the origin of secondary flow from other perspectives (e.g., vorticity), which will be conducted in this paper.

In order to establish the database for the secondary flow, numerical simulations of viscoelastic fluid flow were carried out by commercial computational fluid dynamics (CFD) software ANSYS FLUENT, which can provide mature, reliable, and robust numerical algorithms. However, there is no constitutive model for viscoelastic fluid in this CFD code. The functionalization of user-defined function (UDF) provided by the FLUENT package makes it achievable to embed user-defined scalar (UDS) equations and source terms of governing equations into the calculations. Our previous work [31] has implemented finitely extensive nonlinear elastic-Peterlin (FENE-P) constitutive model into FLUENT by utilizing UDF, and successfully realized the simulation on two-dimensional laminar viscoelastic fluid flow through symmetric planar sudden expansion geometry, which establishes the foundation of the present research. In addition to FENE-P model, the present paper further implements Giesekus model to FLUENT through UDF.

The present paper aims at investigating the mechanism of secondary flow from some new perspectives and the effects of several factors including material parameters and geometrical configuration. Numerical simulations are performed on laminar viscoelastic fluid flows through straight ducts with different cross sections. As an extension of our previous work [31], Giesekus constitutive model is embedded into FLUENT by UDF to describe viscoelastic fluid behavior with non-vanishing N_2. The rest paper is organized as follows: Section 2 introduces the numerical method including computational model, governing equations, UDF-related knowledge and mesh independence; Section 3 validates UDF and analyzes the numerical results, including the origin of secondary flow and effects of material properties and geometrical configuration on secondary flow; finally, main conclusions are drawn in Section 4.

2. Numerical methods

2.1. Computational model

The system considered in this work consists of a straight duct with viscoelastic fluids passing through. The schematic diagrams of the systems are shown in Fig. 1. The origin of a Cartesian reference frame is set at the center of the inlet. The main flow is along the x-axis.

2.2. Governing equations

The flow considered is three dimensional, incompressible, fully-developed, laminar, and isothermal. Hence, the continuity and the momentum equations read as

$$\frac{\partial u_i}{\partial x_i} = 0, \tag{1}$$

$$\rho u_j \frac{\partial u_i}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{\partial \tau_{ij}}{\partial x_j}, \tag{2}$$

where u_i and x_i correspond to velocity components and coordinates, respectively, and subscript $i = 1, 2, 3$ for x, y, z. ρ is the density and set as 1×10^3 kg/m^3, p is the pressure and τ_{ij} is the stress tensor component. For viscoelastic fluid flow, τ_{ij} is presented as,

$$\tau_{ij} = \tau_{ij}^{p} + \tau_{ij}^{s}, \tag{3}$$

where τ_{ij}^{s} is the stress tensor induced by solvent viscosity and τ_{ij}^{p} is the elastic stress tensor, the constitutive equations for the stress tensors are

$$\tau_{ij}^{s} = \mu_s \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right), \tag{4}$$

$$\tau_{ij}^{p} = \frac{\mu_p}{\lambda} \left[f(r) C_{ij} - \delta_{ij} \right], \tag{5}$$

respectively. Herein, μ_s is the solvent viscosity and set as 1×10^{-3}Pa · s, and μ_p is the viscosity of polymer or surfactant solute. λ is the relaxation time, δ_{ij} is Kroneker symbol, $f(r)$ is the Peterlin function and $f(r)$=1 for Giesekus model. C_{ij} is the conformation tensor component of polymer molecules or surfactant micelles, and its transport equation presents as