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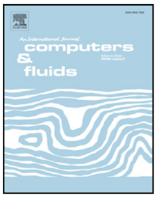

Review

Numerical simulations of steady and unsteady two-phase flows using a homogeneous model

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ABSTRACT

The capability of a homogeneous model to simulate steady and unsteady two-phase flows is investigated. The latter is based on the Euler set of equations supplemented by a complex equation of state describing the thermodynamical behavior of the mixture. No equilibrium assumption is made except for the kinematic equilibrium. The return to the thermodynamical equilibrium is ensured by three source terms that comply with the second law of thermodynamics. The numerical code built on the basis of this model has been verified and some validation results are discussed here. The speed of propagation of a pressure signal is first studied and compared with experimental measurements. Then a more complex situation is investigated: SUPERCANON experiment which corresponds to a sudden depressurization of heated water (associated to a Loss Of Coolant Accident, or LOCA). At last, the results of a numerical experiment of heating of flowing water in a pipe are compared to those obtained with an industrial code.

1. Introduction

When two different phases of the same component coexist in a stable manner, the thermodynamical theory states that their temperatures, pressures and chemical potentials must be equal [1]. Conversely, when the equilibrium between the two phases is not yet reached, these temperatures, pressures and chemical potentials may differ. Most of the models that are used to perform two-phase flow simulations involving mass transfer are based on one or more equilibrium assumptions. This can be restrictive when dealing with highly unsteady flows for which these assumptions may not be relevant. The homogeneous model used in the present work [2–5] only makes the assumption that the velocities of the two phases are equal and the full thermodynamical disequilibrium is accounted for. This hypothesis of equal velocities can show some limitations – for instance when gravity plays an important role, or when simulating jets, etc.... – but it remains acceptable for a lot of industrial applications with forced convection in pipes, as those encountered for nuclear power plants. One can note that some two-fluid models are developed without equilibrium assumptions; for instance those in [6–12] which are based on the Baer–Nunziato model [13], or in [14]. It should also be mentioned that homogeneous models have been studied with different velocities for the two phases [15].

The homogeneous model in the present work was first proposed in [2]. It is built on the Euler set of equations supplemented by three fractions which allow to account for the temperature, pressure and chemical potential disequilibria between the two phases. The return to equilibrium is then ensured by three source terms which are defined in order to fulfill the second law of thermodynamics. The resulting model possesses a complex Equation Of State (EOS) for the mixture and is hyperbolic provided that each phasic entropy is concave with respect to the phasic specific volume and to the phasic specific internal energy, and that the mixture temperature is positive [4]. Moreover, the transition between single-phase flow and two-phase flow is managed quite naturally. This is a crucial feature since a lot of industrial situations involve such transitions.

Among other transient phenomena involving two-phase flows in confined or pressurized devices, one may for instance cite: water hammers [16,17], Boiling Liquid Expanding Vapor Explosion (BLEVE) [18,19], breaches in pressurized pipes, erosion due to the collapse of cavitation bubbles [20], etc.... In such situations, pressure waves are produced that may damage the installation. Hence, the prediction of the propagation of such waves is important for safety studies. It is well-known that the propagation of pressure waves in two-phase flows strongly depends on the gas fraction [21]. Roughly speaking, the speed of sound in a medium depends on the ratio of its specific volume to its compressibility. In a homogeneous mixture of liquid and gas, the compressibility is close to the compressibility of the gas, whereas the specific volume is close to the specific volume of the liquid. As a consequence, the sound

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of speed will be lower than that in the pure phases [22]. This behavior has been investigated with experimental measurements for liquid–gas mixtures (without mass transfer). The aim of the present paper is to evaluate the capability of a code to reproduce some schematic situation encountered in nuclear applications. We first compare the speed of propagation of a pressure pulse for different air–water mixture fractions to the measurements of [21]. A more complex situation of depressurization of heated water is also investigated. In the latter, waves are generated by the sudden depressurization and travel in both the pure liquid and in the steam–water mixture. We also investigate the case of the heating of water flowing in a pipe.

The whole code that is used in this study possesses very few physical parameters, and the time-scale associated to the return to the thermodynamical equilibrium is the most tricky to determine. This time-scale should be chosen on the basis of physical phenomena but this is a difficult task and very few work are available on that subject when considering a complete two-phase flow model [23,24].

The schemes that provide the unsteady numerical approximations are described in [4,25]. An important verification process has been done and it has been reported in [4,25]. In these references, the verification test cases have been chosen to mimic the main situations encountered in the nuclear domain. Two kinds of problems have thus been studied: Riemann problems which are classical unsteady test problems, and a steady-state problem involving the heating of a two-phase mixture. The former (resp. latter) are representative problems of the situation induced by the experiment proposed in Section 6 (resp. 5). The case of Section 5 corresponds to the computation of a steady-state. In order to obtain accurate approximations with little CPU-time, we propose in Section 3 a steady-state algorithm based on the idea of [26].

An overview of the model and the numerical schemes is first proposed in Sections 2 and 3. In Section 4, we assess the computation of the propagation of a pressure-pulse by our code. This is done on the basis of the measurements of the speed of propagation of a pressure-pulse in air–water reported in [21]. Section 5 is then devoted to the study of a case of heating of water in a pipe involving small pressure variations. This case is inspired from the work [27,28]. It is schematic of the steam production in a steam generator of a Pressurized Water Reactor (PWR) and we compare results obtained by the model to the results obtained with an industrial code. We then focus in Section 6 on a more complex situation: the SUPERCANON [29] experiment, which could be assimilated to a two-phase shock-tube with mass transfer. These experiments were conducted in the late 70s on this experimental facility to measure the depressurization of heated water. They were designed to be representative a of a Loss Of Coolant Accident (LOCA) in the primary circuit of a PWR nuclear power plant, as the Edwards'pipe blowdown experiments [30].

2. A homogeneous model for two-phase flows

The aim of this section is to provide a quick overview of the model and of the numerical schemes. For a more detailed presentation, the reader could refer to [2,4,5] for the model, to [4] for the numerical schemes and to [4,25] for the verification process. In the following, a subscript v will denote a vapor or gas quantity, and a subscript l a liquid one.

The quantities describing the mixture of the two phases are: the specific volume $\tau = 1/\rho$ where ρ is the density, U is the velocity and e is the specific internal energy. The three fractions define the way the two phases are mixed in terms of: the volume through the vapor volumic fraction α_v, the mass through the vapor mass fraction y_v and the energy through the vapor energy-fraction z_v. In fact these fractions allow to express the phasic quantities in terms of the mixture quantities:

$$\tau_v = \frac{\alpha_v \tau}{y_v}, \quad \tau_l = \frac{\alpha_l \tau}{y_l}, \quad e_v = \frac{z_v e}{y_v}, \quad e_l = \frac{z_l e}{y_l}, \quad U_v = U, \quad U_l = U, \tag{1}$$

where $\alpha_l = 1 - \alpha_v$, $y_l = 1 - y_v$ and $z_l = 1 - z_v$. The set of equations for the homogeneous model is then:

$$\begin{cases} \dfrac{\partial}{\partial t}(\rho Y) + \dfrac{\partial}{\partial x}(\rho U Y) = \rho \Gamma_Y \\ \dfrac{\partial}{\partial t}(\rho) + \dfrac{\partial}{\partial x}(\rho U) = 0 \\ \dfrac{\partial}{\partial t}(\rho U) + \dfrac{\partial}{\partial x}\left(\rho U^2 + P\right) = 0 \\ \dfrac{\partial}{\partial t}(\rho E) + \dfrac{\partial}{\partial x}(\rho U E + U P) = 0, \end{cases} \tag{2}$$

where $Y = (\alpha_v, y_v, z_v)$, and where E stands for the specific total energy:

$$E = e + U^2/2.$$

The thermodynamical closure of the system is ensured by the definition of the mixture entropy:

$$s = y_v s_v(\tau_v, e_v) + y_l s_l(\tau_l, e_l), \tag{3}$$

where the phasic entropies $s_k(\tau_k, e_k)$ must be specified by the user. The phasic pressures P_k and the phasic temperatures T_k are deduced from the phasic entropies through the phasic Gibbs relations

$$T_k ds_k = de_k + P_k d\tau_k, \tag{4}$$

which lead to the definitions:

$$T_k^{-1} = \frac{\partial}{\partial e_k}(s_k(\tau_k, e_k))_{|\tau_k}, \qquad P_k = T_k \frac{\partial}{\partial \tau_k}(s_k(\tau_k, e_k))_{|e_k}. \tag{5}$$

The pressure law P and the temperature law T for the mixture are obtained by writing the Gibbs relation (6) for the mixture, which involves the mixture entropy (3):

$$T ds = de + P d\tau + \left.\frac{\partial s}{\partial \alpha_v}\right._{|e,\tau,y_v,z_v} d\alpha_v + \left.\frac{\partial s}{\partial y_v}\right._{|e,\tau,\alpha_v,z_v} dy_v + \left.\frac{\partial s}{\partial z_v}\right._{|e,\tau,\alpha_v,y_v} dz_v. \tag{6}$$

We then have the definitions:

$$\frac{1}{T} = \frac{\partial}{\partial e}(s)_{|\tau,Y} \quad \text{and} \quad \frac{P}{T} = \frac{\partial}{\partial \tau}(s)_{|e,Y}, \tag{7}$$

for the pressure and the temperature of the mixture. Using the phasic pressure P_k and temperature T_k, they can be written:

$$P(Y, \tau, e) = \frac{\frac{\alpha_l}{T_l} P_l + \frac{\alpha_v}{T_v} P_v}{\frac{z_l}{T_l} + \frac{z_v}{T_v}} \quad \text{and} \quad \frac{1}{T} = \frac{z_l}{T_l} + \frac{z_v}{T_v}. \tag{8}$$

The three source terms Γ_Y (one for each fraction) rule the thermodynamical exchange between the phases and allow the system to return to the thermodynamical equilibrium. Hence they must be chosen to comply with the second principle of thermodynamics. They are written here as in [2,4]:

$$\Gamma_Y = \frac{Y_{eq} - Y}{\lambda}, \tag{9}$$

where λ is a characteristic time-scale, and

$$Y_{eq} = (\alpha_{v,eq}, y_{v,eq}, z_{v,eq}), \tag{10}$$

defines the equilibrium fractions. This equilibrium state Y_{eq} maximizes the specific entropy of the mixture s for a given specific internal energy e and a given specific volume τ. When the two phases coexist Y_{eq} belongs to $]0, 1[^3$ and the derivative of the entropy with respect to the mixture fractions Y is then null. A simple