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Minimax rule for energy optimization

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ABSTRACT

The purpose of this paper is to present an efficient solution for energy optimization problem. We have identified some similarities between portfolio optimization and energy optimization. Using Kuhn–Tucker conditions we compute the efficient solution for a bi-criteria energy optimization problem.

1. Introduction

This paper is dedicated to Chuck Leith who was a pioneer in numerical simulation and development of predictive modeling capabilities for atmospheric and climate dynamics [16,24]. Challenges associated with a rapidly rising global population – an increase of more than 2.5 billion new urban inhabitants is projected by 2050, relative to 2011 – require high resolution physics-based, coupled, dynamic, and predictive capabilities that not only characterize current multi-scale environmental and socio-economic interactions but also enable the prediction of future impacts within growing cities. Feedback loops and nonlinear interactions interconnect physical and human processes. Understanding of emergent regional climate modifiers (urbanization, energy, water, agriculture) on decadal scales cannot be realized simply by studying these components in isolation. Scenario-based analysis and modeling techniques serve as a new paradigm for integrated studies of regional and urban climate systems on decadal timescales, which are critically important for policy makers [18,25]. Novel computational methods to accelerate and improve accuracy of multi-scale nested models and data analytics are implemented to examine scale dependency of simulated outcomes. The advanced physics-based predictive analytics and statistical modeling tools are utilized to conduct ensemble-based regional hydro climate simulations, focusing on a set of rapidly urbanizing megapolitan areas across multiple climate zones.

Strategic adaptation plans require development to increase production of agricultural commodities, maximize energy and land-use efficiency, enhance community engagement, reduce transportation costs while enhancing profitability, and mitigate adverse impacts such as the urban heat island effect and anthropogenic heating of the urban environment due to air conditioning [25]. Development and refinement of physics based predictive modeling and assessment tools used at fine spatial resolution is necessary to effectively quantify co-benefits and reveal trade-offs prior to any strategy deployment. For example, changes to the worlds electrical power systems and grids threaten to require massive infrastructure investment and cost to power utilities, especially increasing population, built environment and electrical energy demands during peak summertime air conditioning loads, and mismatches between timing of supply and demand due to increases in renewable energy [22]. Brownouts and other grid failures are projected to become more common as peak demands approach grid capacities, with negative economic and public health consequences resulting. Meanwhile a financial barrier exists for the financing of

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grid improvements because utility revenues are proportional to total power sales, whereas utility costs are driven largely by capital and maintenance for the fixed infrastructure. Analyzing daily consumption of energy, fluctuations are visible and even more intense in extreme temperature areas due to summer air conditioning demands. To reduce energy fluctuation and extreme consumption, Ruddell, Salamanca and Mahalov [22] have created a model which enables a partial shift of power demand from peak load, during extreme events such as heat waves.

Large fluctuations/large deviations in electric grids constitute a threat for economic activities. When we analyze energy production, fluctuations have an even bigger impact due to technological complexity of starting/stopping production facilities and storing energy. Reducing daily fluctuation of energy consumption is an objective for the producers, together with the general objective for each company to maximize its profits. Analyzing the entire process from this point of view, we had identified a similarity between energy optimization and portfolio selection problems. Energy fluctuation may be regarded as risk from portfolio selection, while company's result is similar to total wealth.

How do we measure fluctuation/risk and how do we measure company's result/wealth? Let's go back to the origins of portfolio selection. In 1952, Markowitz [19] introduced Mean Variance Model. Variance (l_2 function) is used to measure the risk, while total wealth is calculated as total amount of money cashed in by the investor. The objective is to minimize risk while total wealth does not fall below a specified level or maximize total wealth while risk does not exceed a predefined level. Although Markowitz's paper is considered a milestone for portfolio selection, implementation of Mean Variance Model is difficult due to quadratic form of objective function. Reviewing the literature, we have identified five main directions followed by researchers to extend and improve Markowitz's Mean Variance model: extending the single period model to multiperiod (see Smith [30], Mossin [21], Merton [20], Samuelson [26], Fama [10], Hakkanson [12], Elton and Gruber [8,9], Li and Ng [17]); introduction of transaction costs in the model (see Constantinides [6], Perold [23], Dumas and Luciano [7]); sensitivity of models to input data (see Best and Grauer [1,2], Chopra, Hensel and Turner [5]); developing some approximation schemes (see Sharpe [27–29], Stone [31], Lee, Finnerty and Wort [15], Huang and Qiao [13]); considering new measures for risk (see Konno and Yamazaki [14], Cai et al.[4]). Konno proposed as measure for risk the mean absolute deviation which is an $l_1$function, while Cai et al. measured risk as the maximum over all assets of absolute deviation which is an l_∞ function.

Our paper develops a new approach for energy optimization, based on bi-criteria programming. We employ the risk measure introduced in [4]. These techniques are modified and extended to evaluate the energy fluctuation problem. In Section 2 we formulate a mathematical model for the energy consumption problem. In Section 3 we develop an optimization procedure and compute optimal solutions. This paper concludes with a discussion in Section 4.

2. Problem formulation

An energy plant focuses the problem on determining the optimum quantity of energy to be produced every hour, such that fluctuation of energy during a period of time is reduced to minimum, turnover is maximized and some constraints imposed by the market are fulfilled.

First of all, from the way the problem is defined, it is clear that we are dealing with a bi-criteria problem, where fluctuation has to be minimized and turnover maximized.

Definition 2.1. Fluctuation of energy is the difference between energy produced at a certain hour and a predefined level of energy.

Remark 2.1. Predefined level of energy may be for example a random value chosen by energy plant or the average energy produced during a certain period from the past. Of course, when the average is employed, it may be adjusted with a factor to cover the forcasted demand.

How do we measure fluctuation and turnover? If we think to portfolio selection, we state that risk from portfolio selection and fluctuation from energy optimization are similar and therefore the measures for risk will be valid also for fluctuation. The portfolio selection literature provides, among others, the following measures for risk: variance [19] (l_2 function), mean absolute deviation [14] (l_1 function), maximum absolute deviation [4] (l_∞ function) and conditional value at risk. For our problem, we will employ maximum absolute deviation to measure fluctuation and as predefined level we will use the average. To compute turnover we will refer to a basic economic principle which states that turnover is the total, over entire period of time, of quantity multiplied with price.

Which is the measuring unit for fluctuation and turnover? Regarding turnover, it is clear that it is measured in money (dollars, euro). Fluctuation is referring to energy, which is measured in KWh or multipliers, but does this measuring unit satisfy our problem? To solve the bi-criteria problem, we will transform it in an equivalent parametric problem, which lead to idea that fluctuation and turnover have to be expressed in the same measuring unit. Therefore we have to evaluate fluctuation in money so we will multiply energy fluctuation with price. Introduction of price in the measure of fluctuation is a plus for our optimization process, because price is an element with a considerable impact on demand.

In our paper we are imposing only a simple condition for the energy produced, which means that energy is bounded by the minimum level of energy which the energy plant has to deliver and the maximum production capacity.

Denoting by $1, 2, \ldots, i, \ldots, n$ the time horizon for which the energy has to be optimized and

x_i - energy produced at hour i, $i = \overline{1, n}$,

p_i - price of energy at hour i, $i = \overline{1, n}$,

r - predefined level of energy,

ε - minimum level of energy which the energy plant has to deliver,

ρ - maximum production capacity of the energy plant,

we have the following mathematical expressions for

fluctuation of energy

$$\max_{i=\overline{1,n}} |p_i x_i - p_i r|$$

turnover

$$\sum_{i=1}^{n} p_i x_i$$

constraints

$$\varepsilon \le x_i \le \rho, \ \ i = \overline{1, n}$$

which are determining the following mathematical model for our problem

$$\begin{cases} \min \left(\max_{i=\overline{1,n}} |p_i x_i - p_i r|, \ \ -\sum_{i=1}^{n} p_i x_i \right)^T & \\ \varepsilon \le x_i \le \rho, & i = \overline{1, n}. \end{cases} \tag{2.1}$$

3. Computing the solution

We recall that for a problem

$$\begin{cases} \min f(x) \\ \quad x \in X \end{cases}$$