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Assessment of subgrid-scale modeling for large-eddy simulation of a spatially-evolving compressible turbulent boundary layer

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ABSTRACT

The performance of three standard subgrid-scale (SGS) models, namely the wall-adapting local eddy-viscosity (WALE) model, the Dynamic Smagorinsky model (DSM) and the Coherent Structures model (CSM), are investigated in the case of a spatially-evolving supersonic turbulent boundary layer (STBL) over a flat plate at $M_\infty = 2$ and $Re_\theta \approx 2600$. A high-order *split-centered* scheme is used to discretize the convective fluxes of the Navier–Stokes equations, and is found to be highly effective to overcome the dissipative character of the standard shock-capturing WENO scheme. The consistency and the accuracy of the simulations are evaluated using direct numerical simulations taken from the literature. It is demonstrated that all SGS models require a comparable minimum grid refinement in order to capture accurately the near-wall turbulence. Overall, the models exhibit correct behavior when predicting the dynamic properties, but show different performances for the temperature distribution in the near-wall region even for cases with satisfactory energy resolution of more than 80%. For a well-resolved LES, the SGS dissipation due to the fluctuating velocity gradients is found to dominate the total SGS dissipation.

1. Introduction

Compressible turbulent boundary layer is a basic phenomenon that occurs in a wide range of high-speed applications, such as internal and external aerodynamics of space vehicles. From a physical view-point, this phenomenon is still of primordial interest for fundamental research as well as for numerical modeling.

Highly resolved numerical simulations, using Direct Numerical Simulations (DNS) and Large-Eddy Simulations (LES), emerge as a promising tool to help getting more insight into this topic. A recent literature review on DNS of compressible boundary layers can be found in Shadloo et al. [1]

Few LES of supersonic turbulent boundary-layer flows have been performed [2–5]. For instance, Spyropoulos & Braisdell [2] reported LES of spatially-evolving supersonic turbulent-boundary layer at Mach M = 2.25. A second- as well as a fourth-order accurate upwind biased finite differences schemes were used for the convective fluxes. In terms of wall-units, all considered grids had resolutions ranging between $59 \leq \Delta x^+ \leq 88$ for the streamwise direction, $0.77 \leq \Delta y^+_{min} \leq 0.97$ for the wall-normal direction and $11.4 \leq \Delta z^+ \leq 42.1$ for the spanwise direction. It was concluded that a decrease of about 20% of the computed skin-friction is found when lower order schemes are employed, mainly a third order upwind scheme for the convective terms and second order for the viscous terms. Because of the low considered Mach number, the modeling of the isotropic part of the shear stresses was not found to have a considerable effect on the skin-friction coefficient, C_f. The insufficient amount of turbulent transport was attributed to the use of the dynamic Smagorinsky model, in which the eddy viscosity is computed using the smallest resolved scales.

Using monotonically integrated large-eddy simulation (MILES) approach, Yan et al. [5] have conducted a numerical study of supersonic flat-plate boundary layers in which the numerical dissipation induced by the scheme substitutes the SGS eddy viscosity, mimicking thereby from an energetic view-point the action of the SGS terms on the flow dynamics. The simulated flows evolved at freestream Mach numbers of 2.88 and 4. An adiabatic as well as an isothermal cases with $T_w/T_r = 1.1$ (where the recovery temperature $T_r \simeq T_\infty(1 + r\frac{\gamma-1}{2}M^2_\infty)$, $r = 0.89$ is the recovery factor) were performed. In terms of wall-units, their grid resolutions were $\Delta x^+ = 18$, $\Delta y^+_{min} = 1.5$ and $\Delta z^+ = 6.5$. It was reported that the mean streamwise velocity profiles using the van-Driest transformation were in good agreement with the viscous sublayer linear approximation and law-of-the-wall ($u^+_{vd} = 2.5 \log y^+ + 5.7$). The distributions of the streamwise Reynolds stresses scaled by mean density $\langle\rho\rangle$ and wall shear stress τ_w, were found to be

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very similar except close to the wall, and showed good agreement in the outer region of the boundary layer ($y/\delta > 0.2$). The peak magnitude in the near-wall region ($y/\delta < 0.2$) was supported by both experimental and DNS results, although its location was not consistent with the reference data. The Reynolds shear stresses of both cases showed good agreement with the reference solutions. Finally, the turbulent Prandtl number Pr_t was found to be in good agreement with the experimental value of 0.89.

Urbin & Knight [6] performed an LES of an adiabatic Mach 3 boundary layer. A detailed grid refinement study was performed to assess the required grid resolution in the viscous sublayer, the logarithmic and the outer regions of the boundary layer. They emphasized that the subgrid-scale effects can be modeled using MILES without Smagorinsky model. On the other hand, Kawai & Lele [7] proposed a simple mesh-resolution-dependent dynamic wall model for LES of compressible supersonic turbulent-boundary layer over a flat plate.

Recently, Hadjadj et al. [8] performed a series of LES computations of supersonic boundary layers with detailed statistical analysis of the unsteady flow-field using the WALE SGS model. The focus of their work was on the effects of wall temperature on the near wall turbulence behavior, while the effects of the SGS modeling was left for future investigations. To the authors' best knowledge such investigations related to STBL has not completely been addressed in the literature so far.

In LES, the accuracy of the resolved scales highly relies on the mesh size. Locally refined grids usually lead to more resolved turbulent energy but will definitely be more costly in terms of CPU time and memory requirements. The strategy in LES is then to make the best compromise between accuracy and computational cost. Dissipation of a given SGS model may originate, in different proportions, either from the resolved velocity fluctuations or from the mean-averaged velocity gradients.

The present work aims to assess the prediction quality of three popular SGS models on the near-wall asymptotic behavior of a supersonic turbulent boundary layer (STBL). Some of the turbulence statistics are reported in this paper in order to assess their effects in conjunction with the numerical scheme and the mesh resolution. The obtained results are compared with available direct-numerical simulation (DNS) data and showed an overall good agreement.

The paper is organized as follows: the governing equations and the numerical discretization are presented in Section 2, where the filtered Navier–Stokes equations and the SGS modeling are presented. The results are presented and discussed in Section 3. The issue of the SGS activity is addressed in the same section before drawing the concluding remarks in Section 4.

2. Methodology

2.1. Large-eddy simulation

The filtered compressible Navier–Stokes equations expressed in a conservative form are written:

$$\left.\begin{aligned}
\frac{\partial\overline{\rho}}{\partial t}+\frac{\partial\overline{\rho}\widetilde{u}_i}{\partial x_i}&=0\\
\frac{\partial\overline{\rho}\widetilde{u}_i}{\partial t}+\frac{\partial\overline{\rho}\widetilde{u}_i\widetilde{u}_j}{\partial x_j}+\frac{\partial\overline{p}}{\partial x_i}-\frac{\partial\check{\sigma}_{ij}}{\partial x_j}&\approx-\frac{\partial\tau_{ij}}{\partial x_j}\\
\frac{\partial\overline{\rho}\check{E}}{\partial t}+\frac{\partial\left(\overline{\rho}\check{E}+\overline{p}\right)\widetilde{u}_j}{\partial x_j}-\frac{\partial\widetilde{u}_i\check{\sigma}_{ij}}{\partial x_j}+\frac{\partial\check{q}_j}{\partial x_j}&\\
\approx-\frac{1}{\gamma-1}\frac{\partial\left(\overline{pu_j}-\overline{p}\widetilde{u}_j\right)}{\partial x_j}-\widetilde{u}_j\frac{\partial\tau_{ij}}{\partial x_j}&
\end{aligned}\right\} \tag{1}$$

where $\overline{\rho}$, $\overline{p}$ and $\widetilde{u}_i$ denote density, pressure and velocity vector, respectively. The equation of state, the total energy, the viscous shear stress and the heat flux are given by:

$$\left.\begin{aligned}
\overline{p}&=\overline{\rho}r\widetilde{T}\\
\overline{\rho}\check{E}&=\frac{\overline{p}}{\gamma-1}+\frac{1}{2}\overline{\rho}\widetilde{u}_i\widetilde{u}_i\\
\check{\sigma}_{ij}&=\widetilde{\mu}\left(\frac{\partial\widetilde{u}_j}{\partial x_i}+\frac{\partial\widetilde{u}_i}{\partial x_j}\right)-\frac{2}{3}\widetilde{\mu}\left(\frac{\partial\widetilde{u}_k}{\partial x_k}\right)\delta_{ij}\\
\check{q}_j&=-\frac{\widetilde{\mu}C_p}{Pr}\frac{\partial\widetilde{T}}{\partial x_j}
\end{aligned}\right\} \tag{2}$$

with $\widetilde{\mu}$ is the dynamic viscosity obeying to the Sutherland's law and Pr is the Prandtl number equal to 0.72 (for air with $\gamma = C_p/C_v = 1.4$). Unlike the $\overline{(.)}$ and the $\widetilde{(.)}$ symbols, the $\check{(.)}$ symbol does not denote a filter operation but indicates that the quantity is based on primitive filtered variables. Thus, $\check{E}$ refers to the resolved total energy, which is not equal to the filtered total energy. Note that, following [9–11], the unclosed sub-grid scale (SGS) terms are neglected in both momentum and energy equations.

In this study, different LES models are used to model the action of the subgrid-scales (SGS) on turbulence: the Dynamic Smagorinsky procedure of Germano et al. [12], Moin et al. [9] and Lilly [13], the coherent structures model proposed by Kobayashi [14] and the Wall-Adapting Local Eddy-viscosity model by Nicoud and Ducros [15]. In order to better quantify the real contribution of the SGS modeling, an Implicit LES is also performed, in which the numerical dissipation mimics the action of the small scales on turbulence.

2.1.1. Modeling the SGS stress tensor

The SGS stress tensor, τ_{ij}, in Eq. (1) is defined by:

$$\tau_{ij}=\overline{\rho}\left(\widetilde{u_iu_j}-\widetilde{u}_i\widetilde{u}_j\right) \tag{3}$$

It is modeled via the definition of a SGS eddy viscosity, μ_{sgs}, which yields:

$$\tau_{ij}-\frac{1}{3}\tau_{kk}\delta_{ij}=-2\mu_{sgs}\left(\widetilde{S}_{ij}-\frac{1}{3}\widetilde{S}_{kk}\delta_{ij}\right) \tag{4}$$

where $\widetilde{S}_{ij}=1/2\left(\partial\widetilde{u}_i/\partial x_j+\partial\widetilde{u}_j/\partial x_i\right)$ is the strain rate tensor of the resolved scales. The SGS viscosity, μ_{sgs}, is given by:

$$\mu_{sgs}=\overline{\rho}C_s\Delta^2|\widetilde{S}| \tag{5}$$

where $|\widetilde{S}|=\sqrt{2\widetilde{S}_{ij}\widetilde{S}_{ij}}$ is the second invariant of the strain rate tensor, and C_s is a dynamically-retrieved modeling constant.

For compressible flows, Yoshisawa [16] proposed a closure for the isotropic part of the SGS stress tensor, τ_{kk}, defined by:

$$\tau_{kk}=2\overline{\rho}C_I\Delta^2|\widetilde{S}|^2 \tag{6}$$

The model constant, C_I, is dynamically retrieved for the DSM procedure, or set equal to 0.005 for the CSM (Moin et al. [9]). Unless stated, the isotropic part of the SGS stress tensor, τ_{kk}, is not modeled for both CSM and WALE model.

Dynamic Smagorinsky model

In the Dynamic Smagorinsky procedure, the model's constants, C_s and C_I, are dynamically extracted from the resolved flowfield quantities. A *test* filter, denoted as $\widehat{(.)}$, whose width is larger than the grid-filter width, is applied to the grid-filtered quantities. The model's constants are then calculated at the *test*-filter wavenumber, and are assumed to remain about the same within $[k_{test}, k_c]$ wavenumbers range. Denoting $\widehat{\Delta}$ as the *test*-filter width and Δ is the grid-filter width, it is common to define $\widehat{\Delta}/\Delta = 2$.

After dynamically retrieving C_s and C_I, and to avoid any numerical instability due to negative values, both constants are averaged